

Exemplars[®]

How many pebbles each boy has.
 Under each boy on a table and do one
 find a rule
 half of 75 is about 37 so I
 guess. I know you can only use
 you don't get a remainder.

COLLECTING PEBBLES

pebbles for matt	pebbles Total
51	85

$$\frac{1}{2} \text{ of } 34 = 17$$

$$\begin{array}{r} 34 \\ + 17 \\ \hline 51 \end{array} \quad + \frac{34}{85} \text{ too high}$$

Key
 n is Cameron's
 pebbles

$$\frac{1}{2}n \quad n + \frac{1}{2}n = 75$$

$$\frac{1}{2}(34) \quad 34 + \frac{1}{2}(34) = 75$$

$$34 + 51 \neq 75 (85)$$

too high so I need to go lower.

$$\begin{array}{r} 16 \\ 2 \overline{)32} \\ \underline{32} \\ 0 \end{array} \quad \begin{array}{r} 16 \\ 2 \overline{)32} \\ \underline{32} \\ 0 \end{array}$$

try 32
 $\frac{1}{2}n = 75$
 $\frac{1}{2}(32) = 75$
 $2 + 48 \neq 75 (80)$
 was too high again. so I will try 30.

$$\begin{array}{r} 15 \\ 2 \overline{)30} \\ \underline{30} \\ 0 \end{array} \quad \begin{array}{r} 30 \\ 2 \overline{)45} \\ \underline{45} \\ 0 \end{array}$$

Answer
 Cameron 30 pebbles
 Matthew 45 pebbles

Getting Started:

A Guide to the
 Successful
 Use of
 Exemplars

Math
 K-12

Getting Started

This guide will provide you with an individualized approach to using *Exemplars Math*. *The Guide to Getting Started* is an effective staff development tool, that can be used alone or in a workshop setting.

This step-by-step tutorial for using the *Exemplars Math* program helps your students use performance assessment tasks that reflect state, national and Common Core standards in mathematics. You will gain an understanding of how to use the *Exemplars* activities with your students. We did not bind the guide, so you could easily make copies to share with your colleagues. You can also download additional copies from *Exemplars* Web site, please visit http://www.exemplars.com/assets/files/the_guide.pdf.

For more information: Please call 800-450-4050; send an e-mail to info@exemplars.com; and be sure to visit our **Web site** at www.exemplars.com, where we continue to add new material. We also encourage you to participate in our blog at www.exemplars.com/blog. Engage in the conversation and let us know what issues are important to you and what your needs are. We look forward to hearing from you.



Exemplars tasks can serve as instructional tools as well as assessment tools.



Getting Started cont.

In order to help administrators and teachers meet their growing achievement needs, *Exemplars* has aligned all of its material (Math Pre K–12, Science K–8, Reading, Writing and Research 5–8 and Developing Writers K–4) with state, national and Common Core standards, as well as grade-level expectations. We continue to align our material with a variety of curriculum and resources, to make the implementation of *Exemplars* easier in your classroom. To access our current alignments please visit: <http://www.exemplars.com/resources/alignments>

Math alignments include:

- Common Core Standards
- State Standards
- NCTM Standards
- National STEM Standards
- Everyday Mathematics
- Investigations
- Impact Math

The successful experience of *Exemplars* users and workshop participants is reflected in this guide. For more professional development ideas please visit www.exemplars.com. *Exemplars* customizes professional development workshops to meet the needs of individual schools and districts. Please contact us to arrange professional development for your teachers or for product information regarding Pre K–K Math; K–8 Math; Secondary Math; K–8 Algebra; K–8 Science; 5–8 Reading, Writing and Research; K–4 Developing Writers and K–5 Spanish Exemplars and our tools for the differentiated classroom.

Exemplars Math Guide

Table of Contents

Introduction	5
Quick Start	6
School/Teacher Tips	7
Using <i>Exemplars</i> in the Classroom	9
Problem Solving	13
Assessing Student Work	14
Examining Student Work	16
Formative Assessment	17
Using Technology	19
<i>Getting Started: A Teacher's View</i>	20
<i>Using Journals in Mathematics</i>	22
<i>Introducing Rubrics to Students</i>	26
Exemplars Rubrics	31
Sample Task, A Puzzle	39
<i>Exemplars</i> Rubric Scoring Notes	49
Task Graphic Organizer	50
Preliminary Planning Sheet	51
KWL Chart	52

For more information please contact us via e-mail
at info@exemplars.com, or call us at 1-800-450-4050.

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Introduction

Exemplars can be used...

- To help **instruct** your students in problem solving and applying concepts in meaningful, real-life situations
- To **explore** how your class is performing in **problem solving** and give you an overall feel of your students' skills
- To **diagnose** the ability of particular students to apply and use math concepts and **solve problems**
- To help your students learn to **self-assess** their mathematical skills using the rubrics and anchor papers
- To help your students **communicate** mathematically by sharing their thinking process verbally and in written form



The key is to listen and learn about the thinking students use when approaching tasks.



Exploring how your classroom is performing; diagnosing students' abilities to apply math concepts; helping your students self-assess; and using that information to alter instruction and learning, all adds up to effective **formative assessment**.

Quick Start – Read This Page!

Many teachers find it useful to select a few *Exemplars* tasks to teach their students strategies for problem solving. They can also use tasks for evaluating students' problem-solving skills.

An *Exemplars* task includes four parts:

1. The first page highlights the problem to be given to the students. It is large enough to make a transparency or simply photocopy it to distribute to students.
2. There are rich teacher notes designed to help you prepare for the problem. Differentiated material is included (for grades Pre K–8), offering a more challenging and a more accessible version and solution for each task.
3. Each lesson has a task-specific rubric at four levels describing what a student would do at each level of performance; Novice, Apprentice, Practitioner and Expert.
4. Anchor papers (examples of student work at each level of performance) are included with each task. These are annotated to focus on particular aspects of student performance. These may also be used for student self- and peer-assessment.

School Tips

- To get students accustomed to the rubric and the expected performance criteria, give them an *Exemplars* task to practice, then share the anchor papers on overhead slides. Discuss and critique them, and score them as a class.
- If a task is not going well or it seems too hard, leave it. Do not use it as an assessment piece, but use it as an instructional piece, and solve it together. It is important for students to see a model of what the teacher is looking for in a response, and you will already have gained important assessment information.
- If you are concerned students are going to be uneasy about getting started, conduct a **group brainstorm** ... and/or a 15-minute **conference**. Students can share ideas, strategies, and ask questions of each other before getting started. The brainstorming might include; identifying different approaches students might use, information that needs to be known, and types of mathematical representations they would use to communicate results.
- One task may not work for all students. With our differentiated material we have included a more challenging and a more accessible version of each task, and you can use the CD-ROM or the subscription's digital library CD and modify/personalize the tasks further by using the "cut" and "paste" editing tools. Please see page 19 for more "Using Technology" tips.

For a closer look at an *Exemplars* classroom please refer to the piece written by Stacey Dement, a first grade teacher in Converse, Texas, describing how she got started with *Exemplars*. (p. 20)

School Tips cont.

- Some schools assign one person to be “in charge” of distributing *Exemplars* tasks. This person might provide a monthly preview of available tasks at faculty meetings or copy tasks s/he thinks might complement the work currently being done in classrooms.
- Other schools separate the hard copy *Exemplars* tasks into three binders; Grades K–2, Grades 3–5 and Grades 6–8 to make sharing resources easier.
- Some schools copy and file *Exemplars* tasks by the content area addressed.
- Make *Exemplars* as user-friendly as possible for the maximum number of teachers and staff in your site-licensed school by sharing or copying the CD on your computer server.
- Use *Exemplars* in parent conferences. Let students lead parents through the problems. Have parents work with the problems to see what kind of mathematics their children are doing. Ask parents to show what arithmetic skills are required.
- Introduce the *Exemplars* student rubrics to your students. Students can keep copies of the rubric, and teachers can put up a large rubric chart on the wall so that students can refer to it as they wish.
- Teachers can start an *Exemplars* journal to help them document their personal experiences as they learn to use *Exemplars*.
- Students can record their “math thinking” in journals while solving *Exemplars* tasks. On page 22 we have included the article, *Using Journals In Mathematics*, written by Lori Jane Dowell Hantelmann, an Elementary Mathematics Specialist for the Regina Public Schools in Saskatchewan, Canada.

Using Exemplars in the Classroom

When planning units we recommend using the backwards design process (see example on page 11) as a means to assist the teacher with ensuring that units of study are aligned with local, state, national or Common Core mathematical standards. The process is as follows:

- 1. Select Standards:** These are the standards that you will assess during the course of the unit. It is important to choose a balance of content and skill standards for the unit. It is also important to limit the number of standards you select to three-five total for a typical four-week unit of study. Select standards that embrace important ideas and skills for the students at your grade level and for the topic you are teaching.

In our subscriptions, each of the *Exemplars* tasks has been plotted on a matrix that shows its relationship to the National Council of Teachers of Mathematics Standards. Each task has also been keyed to arithmetic skills. If you have one of our Best of Math CD-ROMs, you can search for tasks containing the specific standard you wish to teach. You can select tasks that meet the standards and cover specific skills. This will help you keep problem solving and skills in balance.

How to Access Exemplars Alignments: In order to help administrators and teachers meet their growing achievement needs, *Exemplars* has aligned all of its material (Math Pre K–12, Science K–8, Reading, Writing and Research 5–8 and Developing Writers K–4) with state, national and Common Core standards, as well as grade-level expectations. We continue to align our material with a variety of curriculum and resources, to make the implementation of *Exemplars* easier in your classroom. To access our current state and product alignments please visit: <http://www.exemplars.com/resources/alignments>

- 2. Build Essential Questions:** Essential questions address the Big Ideas, Concepts, Skills and Themes of the unit. These questions shape the unit; focus and intrigue students on the issues or ideas at hand; and are open-ended with no obvious right answer. These questions should be important and relevant to the students and allow for several standards if not all of the standards selected to be addressed. These questions should engage a wide range of knowledge, skills and resources and pose opportunities for culminating tasks or projects where students can demonstrate how they have grappled with the problem.



Exemplars tasks are powerful, culminating assessment activities.

3. Select Design Culminating Tasks: This final task or project should encompass and help assess each of the standards selected and should enable students to answer or demonstrate understanding of the answer to the essential question. The task should be multi-faceted, allow for multiple points of entry and be performance based. It should allow students to apply their skills and knowledge learned in meaningful and in depth ways. *Exemplars* tasks that match the standards selected can be powerful culminating tasks. Consider what criteria you will use to assess student learning both before, during and after the unit.

4. Develop Learning and Teaching Activities: These activities and tasks should address the standards selected and guide student learning towards what they need to know and be able to do in order to achieve the standards. Select relevant *Exemplars* tasks that assist with teaching appropriate content, skills and/or problem-solving strategies. There are three **major types** of learning and teaching activities:

- **Introductory Activities** are used to pre-assess students' prior knowledge and to generate student interest in the unit of study. These activities tend to be interactive, exploratory and stimulating.
- **Instructional Activities** are used to provide opportunities for students to learn and demonstrate specific skills, knowledge and habits of mind. These are usually sequenced and scaffolded, tied to specific standards and evidences, interesting, engaging, in-depth, active and interactive. **These activities can also be used for formative assessment during the course of the unit to measure student progress.**
- **Assessment Activities** and the Culminating Activity are used to assess both students' progress towards attainment of the standards and for summative purposes at the end of the unit. These activities usually involve some type of product or performance by the student.

* *All activities selected, both Exemplars tasks and other activities, should be based upon their utility in helping students learn and demonstrate the knowledge and skills identified in the standards selected. Activities should accommodate a range of learning styles and multiple intelligences and be developmentally appropriate. Activities should also have a purposeful and logical progression for both knowledge and skill attainment.*

5. Assess Student Products and Performances. Use the *Exemplars* rubric to assess relevant knowledge, skills and problem-solving strategies as students work on and complete *Exemplars* problem-solving tasks. Collect and use examples of student work that demonstrate the criteria selected and the different levels of performance. Allow opportunities for students to self-assess using the *Exemplars* rubric or one of our student rubrics.



Exemplars tasks are great for instruction AND assessment.

An Example of the Backwards Design Process

Standards

NCTM Content Standards and Evidences (Grades 3–5):

Understand meanings of operations and how they relate to each other.

- Understand various meanings of addition, subtraction, multiplication and division.
- Develop fluency in adding, subtraction, multiplying and dividing whole numbers.

Understand patterns, relations and functions.

- Describe, extend and make generalizations about geometric and numeric patterns.

Essential Question: How does understanding patterns help us to solve mathematical problems?

Culminating Task (Grades 3–5): *A Puzzle* (p. 39)

For the culminating activity the teacher may choose to use the regular task, the more accessible version or the more challenging version. To use this task as a culminating activity, students will also be asked to do three things: draw and/or represent their solution in some form, discuss how they used their knowledge of fractions to solve the problem, and communicate their learning to the class via a class presentation or demonstration.

Learning and Teaching Activities

Introductory Activities: These might include a KWL (Know-Want to Know-Learned) chart (p. 52), exploration of pattern blocks to build a variety of patterns, and recognizing numeric patterns as well as other patterns in the world around us.

Instructional Activities: These might include more focused activities around building different patterns using pattern blocks, numbers and other materials. Activities could also include having students represent the patterns they create in mathematical form. Some *Exemplars* tasks to use for instruction include:

- Bug Watching (K–2)
- Frogs and Flies (K–2)
- Pattern Block Walls (3–5)
- Checkerboard Investigation (3–5)
- Tower Problem (3–5)
- Batches of Beavers (3–5)
- Day Lily Dilemma (6–8)
- Bridges (6–8)
- Building Block Dilemma (6–8)

Assessment Activities: During the course of the unit select two or three problem-solving activities to use for assessment purposes. These will help inform instruction by providing information about how students are progressing towards the standards and about their problem-solving abilities. Students can self-assess, and then revise and edit their solutions to the problem.

Products and Performances: Student products and performances will include all work done when problem solving; as well as other work done around pattern identification and extension.

Tools for Planning and Implementation

Task Organizers and Preliminary Planning Sheets

Task organizers and/or preliminary planning sheets can be invaluable resources for teachers. They are used to anticipate what a student might do on a particular task, and enable the educator to foresee the teaching that should be done before the task is used for assessment.

These planning tools can be easily integrated into the backward design process. They ask the educator to think about the underlying mathematical concepts to anticipate, what strategies and reasoning the student might use to solve the task and their possible solutions. These also encourage the educator to consider the mathematical language students might use in their solution as well as the mathematical connections they might make. The latter two are particularly important because these areas are often overlooked in problem solving – yet, they are a central part of the NCTM standards.

Please see samples of each on page 50 and 51.

Problem Solving

Listed below are a set of problem-solving steps that we recommend teachers use with their students. These steps can be posted and taught to students when using *Exemplars* tasks for instructional purposes. When beginning problem solving with students it is important to model this process and go through it a number of times as a whole group until each student feels comfortable using them independently. Teachers have also printed these steps for students to keep in their problem-solving folders and refer to each time they start a new problem. It is always a good idea to teach students how to use different problem-solving strategies for solving problems (step #4 below) such as drawing a picture, working backwards or solving a simpler problem. It is also important to keep in mind that students have many of their own creative strategies for problem solving. Encourage them to use these strategies.

Problem-Solving Steps

1. Read the problem.
2. Highlight the important information.
3. What do you know and need to find out?



4. Plan how to solve the problem: what skills are needed, what strategies can you use, and what ideas will help you?
5. Solve the problem.
6. Draw and write about your solution and how you solved the problem.
7. Check your answer.
8. Share a connection or observation about this problem.
9. Be sure to provide your students with a copy of the rubric or use one of *Exemplars* student rubrics. We publish student rubrics that are written in language more easily understood by younger students. Take a look at the examples on pages 27–38 of this guide. Visit our Web site www.exemplars.com for more examples.
10. What instructional materials/ technology manipulatives will you need?



In the beginning, it is important to model the problem-solving process repeatedly with students as a whole group until each student feels comfortable using it independently.

Assessing Student Work

The Rubrics

An exciting component of the *Exemplars* program is our **rubrics (pgs. 31–33)**. These scoring rubrics allow you to examine a student’s work against a set of desired learner responses (analytic assessment criteria), and determine where the student is on a developmental continuum. Students may use the rubric to self-assess.

Before You Begin – Familiarize Your Students

In order to be successful, many students need to first understand the rubric that is being used to assess their performance before it is implemented. To facilitate this, we recommend that teachers introduce the idea of a rubric to their students by collectively developing one (or several) that does not address a specific content area, but rather other areas of performance, quality or self-evaluation.

For suggestions on getting started with rubrics and/or sample “non-content area” rubrics, refer to the piece on page 26 called, *Introducing Rubrics to Students*, written by Deb Armitage, Exemplars consultant.

Exemplars Classic 3-Level Rubric Criteria

The *Exemplars Classic Rubric (p. 31)* was used for the first eight volumes of the subscription as well as for the *Best of Math Exemplars* CD-ROMs. It uses three analytic assessment criteria, that are located along the top of the rubric.

- The first, **Understanding**, asks the question, “How well did the student understand the problem?”
- The second, **Strategies, Reasoning and Procedures**, draws your attention to the strategies the student uses in solving the problem. It asks whether or not there is evidence of mathematical reasoning, and points to the appropriate application of mathematics procedures the child uses in solving the problem.
- Finally, **Communication** focuses on both the student’s own explanation of his/her solution, and the use of mathematical representation, terminology and notation.
- **Levels of Performance:** Along the left column of the rubric, you will see the four levels of performance that are used to describe what type of learner strategies and understandings each student exhibits as s/he completes each task. Read through the descriptions of the Novice, Apprentice, Practitioner and Expert categories to understand the distinctions between each.

Exemplars Classic 5-Level Rubric Criteria

Exemplars current **Standards-Based Rubric (pgs. 32–35)** has been modified to reflect revisions in the NCTM Process Standards. These process standards highlight ways students should acquire and use the five content standards (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability), and are more specifically defined than in the past.

The levels of performance — **Novice, Apprentice, Practitioner and Expert** — have not changed. The criteria for assessing student performance parallel the NCTM process standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation). Even though there are more criteria, five compared to three in the original rubric, the language of the rubric is more precise and will make assessment easier. There is also a glossary of terms used in the rubric to help teachers as they assess.

The revisions to this rubric make it easier for teachers to pinpoint students' specific strengths and weaknesses in the problem-solving processes, providing more diagnostic information to teachers in assisting students to meet high standards. Of course if you prefer, you may still continue to use the *Exemplars* Classic Rubric.

Anchor Papers

In assessing student work, it helps to select *anchor papers*. These are examples of previously scored student work at each level of performance. This will make your assessment more efficient. The anchor papers follow each *Exemplars* task.

Remember that anchor papers are guides to be used with judgment. At times, you may need to step back to see where a particular student's solution fits.



Please note: Do not panic if each of the four levels of performance (Novice, Apprentice, Practitioner, Expert) are not represented in a particular problem. It is possible to not have any Novices, or in some cases, there may not be any Experts. Your students may have a great deal of experience concentrated in one performance area.

Examining Student Work in the Classroom

After the students have worked with the task, you can use the *Exemplars* scoring rubric to look at student work. But first you should look at the work and make your own observations about what you see.

1. **Sort** the work into groups by level of performance: Novice, Apprentice, Practitioner or Expert.
2. **Describe** what the work in each group has in common:

Novice:

Apprentice:

Practitioner:

Expert:

3. **Return** to the *Exemplars* scoring rubric and see if your descriptions fit with the *Exemplars* descriptions.
4. **Examine** any discrepancies in scoring and try to understand what causes them. Write some of your thoughts: (Please refer to our rubric scoring notes located in the back of this guide).

Formative Assessment

Assessment That Improves Performance

Exploring how your classroom is performing, diagnosing students' abilities to apply math concepts, helping your students self-assess, and using that information to alter instruction and learning, all adds up to effective **formative assessment**. Formative assessment's sole purpose is to improve student performance – in comparison to **summative assessment**, which focuses on accountability and is used to label, sort or classify students' performance.

The conditions for successful formative assessment include:

1. The student and teacher share a common understanding of what constitutes quality work. That is, they have the same standards for achievement.
2. Both student and teacher can compare the student's performance to the standards that have been mutually set.
 - The student self-assesses as s/he is working on the task **and** upon completion of the task.
 - The teacher may assess the student's work while it is in progress or when completed.
3. Teaching and learning activities are adjusted, so the gap between the standard and the performance is closed.
 - After the teacher assesses the student's performance, they will provide feedback (guidance), enabling the student to improve her/his performance.
 - The teacher also assesses the instruction that preceded the performance. The teacher will adjust future instruction based on this assessment.
 - The student will use what they have learned from the assessment to improve future performances.

Helping students become effective self-assessors has an enormous impact on student performance. Studies by Paul Black and Dylan William^{1,2} show that effective classroom assessment has a greater impact on student achievement than any other approach.

Using Exemplars to Implement Formative Assessment in Your Classroom

Exemplars is designed to help teachers integrate formative assessment into their classrooms. Each math task provides information that students can use for self-assessment.

Annotated Anchor Papers and Rubrics

All of our material includes anchor papers (student work samples), general assessment rubrics and task-specific rubrics. These tools enable teachers to **define** for their students what type of performance is expected on lessons throughout their units of study. Students' peer- and self-appraisals encourage them to **compare** their work using student rubrics and annotated anchor papers. This process demonstrates success as well as offers opportunities for refinement. Based on student performance, teaching and learning strategies can be **adjusted**.

Assessment and Teaching Strategies Tied to Successful Formative Assessment

According to Wiliam and Black^{1,2} the assessment and teaching strategies closely tied to successful formative assessment are:

- **Effective Questions** - Ask meaningful questions, increase wait time for student answers, and have rich follow-up activities that extend student thinking. "Put simply, the only point of asking questions is to raise issues about which a teacher needs information or about which the students need to think." ² (13)
- **Appropriate Feedback** - Black and Wiliam found that giving grades does not improve performance. Using tasks and oral questioning that encourages students to show understanding, providing comments on what was done well and what needs improvement, and giving guidance on how to make improvements should be focused on instead of grades.
- **Peer- and Self-Assessment** - Peer- and self-assessments "secure aims that cannot be achieved in any other way." ² (15) Achieving success requires that students have a clear understanding of the standards, and that they be taught the skills of peer- and self-assessment.

1. Black, Paul and Dylan William, *Assessment and Classroom Learning*. Assessment in Education, Vol. 5, No. 1, 1998. Carfax Publishing Ltd.
2. Black, Paul and Dylan William, *Working Inside the Black Box: Assessment for Learning in the Classroom*. Phi Delta Kappan, September 2004.

Using Technology

If you have received an *Exemplars* subscription, you have received a print version as well as a digital library on a CD-ROM. If you have received a *Best of Math Exemplars* CD-ROM you have received only a CD.

Subscription

The CD in the subscription is a digital archive. It contains exactly the same material that you will find in the print copy of the subscription. You may copy it to your hard drive, put it on your network to share with any teacher in your site-licensed school, or print any task from the PDF files.

Best of Math Exemplars CD-ROM I, II or III

The *Best of Math Exemplars* is a browser-based, searchable CD-ROM that contains performance tasks. You can search for particular problems by standard and grade level, by clicking on a particular NCTM standard and the appropriate developmental level — K–2, 3–5 or 6–8. The tasks that correlate with that standard and developmental level will appear. You can copy the *Best of Math Exemplars* to your hard drive or put it on your site-licensed school’s internal network.

Altering Tasks

While each *Exemplars* CD offers a differentiated version of each task, you may still wish to revise or alter a task. You might want to make a problem more challenging or more accessible for your students by changing the numbers or wording. Or, you might want to personalize a task with names and places of things that students are familiar with. With either CD, you can highlight the words in the Adobe® Acrobat PDF file and copy to a word processing program using the “text” tool.

Congratulations! You have just been introduced to one of the most effective ways teachers across the country are using *Exemplars* in their classrooms! Our activities will give you important information about your students as you become more aware of how they think.

This is the beginning of developing a reliable scoring system. Reliability means that colleagues will apply consistent standards to their students, and therefore all students will receive the same message about what constitutes good work.

Success Stories

Getting Started: A Teacher's View

Stacey Dement, 1st Grade Teacher, Eloff Elementary, Converse, Texas

"Oh, No! Not another pilot program!" I heard mumbled from the back of the library. I guessed mine were not the only eyes rolling toward the ceiling. The *Exemplars* program is designed to assess students' problem-solving and mathematical communication skills. It also supports higher-level thinking and extension of mathematical reasoning.

At a loss about where to begin, I began planning in my mind. How could I begin teaching mathematical communication when many of my students had problems communicating their ideas in writing? I decided I had to start with the class as a whole group, modeling the behaviors and expectations I wanted students to utilize. Then I planned to continue breaking the whole class into smaller groups for successive problem-solving experiences. This "plan of attack" allows them to gradually take more responsibility for problem solving and communication. Only after modeling and allowing large and small group practice would students be assessed independently.

I started by discussing the assessment procedures, rubric and holistic scoring associated with *Exemplars*. I explained the levels of Novice, Apprentice, Practitioner and Expert to my students and told them the expectations for each. We created a poster for the classroom explaining the scoring rubric in "kid words" that all the students could understand and remember. We posted it on our *Exemplars* bulletin board for reference during the year. I also provided an "exemplary word wall" for the students' use when writing. During the year, as we encountered new mathematical terms, we added a word card for each term to our word wall. This was an invaluable tool for the students. I made it a point to send information on *Exemplars* to parents in our newsletter, and I always provided families with an *Exemplars* problem based on the concept we were working on in class. Many students reported enjoyment at working with family members.

I introduced the first three or four *Exemplars* tasks to the whole class on the floor. As a class, we would read the task and analyze exactly what we were being asked. We would then discuss and decide how best to solve the problem, recording any differences in opinion.

Following those same basic steps, I broke the class into small groups to solve their *Exemplars* problems. Then... I became invisible. I was only allowed to observe, facilitating group cooperation if necessary. This forced both small and partner groups to work more independently.

After two whole-class, four small group and two partner *Exemplars*, I felt my students had enough experience to begin independently solving their problems. I watched as each student improved in one way or another. My students became eager to work on *Exemplars*.

After a year of teaching the program I look back and don't have to roll my eyes. I discovered that *Exemplars* was not a program after all, but an assessment philosophy with valuable tools for students and teachers. Implementing *Exemplars* was not difficult, but implementing it incrementally made all the difference. Even young students can become independent problem solvers and learn to communicate their reasoning and mathematical thinking processes through open-ended *Exemplars* tasks. The key is in working incrementally with students, allowing them to feel successful with group experiences before being assessed independently.

Success Stories

Using Journals in Mathematics

*Lori Jane Dowell Hantelmann, Elementary Mathematics Specialist,
Regina Public Schools, SK Canada*

My teaching philosophy has changed significantly since I began teaching in 1990. My students sit at tables and mathematical concepts are taught in a problem-solving context instead of being presented in isolation through discrete workbook pages. One change in my teaching is due to the classroom research into mathematics journals I completed recently for my Masters of Education. My students became active participants in their own learning as they confidently solved realistic problems and explained their ideas in mathematics journals, and I was excited to be part of this rewarding learning environment.

In past years, when attempting to have children write in a mathematics journal, I would read: “This was easy. I like math.” My students were not able to successfully reflect or share what they understood about problem solving or mathematics through their writing. Frustrated, I began to read about using math journals in the classroom.

What is a math journal?

My students write in a notebook to answer open-ended questions using numbers, symbols, pictures and words, and their writing can best be described as written conversations. A math journal is a place where every student has the opportunity to verbalize their math knowledge to their teacher, internally to themselves, and to their classmates. Students’ writing becomes a source for social interaction as they read journal entries to partners and the whole class, talk about their learning and listen to others share different levels of mathematical reasoning.

What did I learn about using math journals?

My classroom research on mathematics journals lead me to recognize four important steps needed to help students write reflectively about mathematics.

1. Teachers need to model the writing process and the language of mathematics. First I modeled my own problem-solving process by thinking out loud as I solved problems and as I recorded my reflections on chart paper. Students soon began to contribute their own ideas about problem solving but continued to model the writing process by recording their comments on chart paper. Students copied these sentences into their math journals. Modeling the writing process took longer than I expected as students needed to become familiar with reflective writing and the language of mathematics, the words and symbols unique to mathematics. Once students became familiar with the vocabulary necessary to communicate in mathematics they began to independently express their own thoughts on paper.
2. Teachers need to ask open-ended questions to guide students in their writing. I learned how to ask open-ended questions to help students think about their own understanding of problem solving and to guide their writing. I began my research by using a list of questions I found in *Writing to Learn Mathematics*, by J. Countryman (1992). Students answered my questions verbally at first and became comfortable sharing their thoughts and ideas with others. It was through their participation in our verbal discussions that students learned how to reflect upon their own knowledge of mathematics and to record their ideas on paper. I soon adapted the questions I found to better meet the needs of my students and to match the problems we were solving. Here are examples of questions I used in my research:
 1. Why was this problem easy?
 2. Would this problem be easier today than yesterday? Why?
 3. What did you do to solve this problem?
 4. Are numbers important in solving this problem? Why?
 5. Did graphs help you to solve the problem? Why?

Using Journals in Mathematics *cont.*

3. Students need to revisit similar tasks to increase their confidence as problem solvers and their knowledge of problem solving. As a teacher of young children, I quickly realized that involving students in rewriting similar problem-solving tasks to the problems they just solved was important in developing their confidence as problem solvers and in understanding the process. Children could not always solve the task independently the first time and were enthusiastic to help rewrite the task and solve it again. I noticed they were more successful in solving the second task.

For example, we solved the task “Space Creatures” (*Exemplars*, 1996, March): On a new planet the astronauts discovered unusual creatures. The features they counted were 15 eyes, 3 horns, 12 legs and 7 arms. More of the creatures had scales than fur on their bodies. Draw your creatures and make a graph for each creature’s features.

The next week, we revisited a similar problem by writing our own version of “Space Creatures” called Super Robots: Students were visiting a robot factory. They saw and counted 13 eyes, 10 legs, 8 knobs, 9 arms and 4 antennas. More bodies were triangles than rectangles. Draw a graph first for each of your robots and then draw a picture of your robots to match your graphs.

4. Teachers need to support students in recognizing their individual problem-solving styles. Students discovered they were different in the strategies they used to solve problems. I guided students to solve “Space Creatures” by drawing pictures of the creatures before making their graphs. For Super Robots, I asked students to make their graphs before drawing their pictures.

Some of the students became frustrated when I asked them to begin their problem-solving task with a graph. I was curious to know why, so my first journal question asked students: “Is it easier to draw a picture first or draw a graph first?” Twelve children chose to draw their picture first and 11 children chose to create their graph first. Surprised with the split in their choices, I asked students to explain: “Why do you think it was easier to draw the graph or picture first?” The children wrote:

The picture was easier because you could count the objects better than the graph. (mathematics journal, Oct. 23, June)

It’s easier to draw the graph first. Yes, because I knew what my Robot would look like. The graph helped me to count the features. (mathematics journal, Oct. 23, Cassy)

Their responses led me to realize the importance of drawing pictures in the problem-solving process for some children, but not all children. I needed to listen carefully to students so I would know how to best support them in recognizing their individual needs as problem solvers.

For four months I had supported students in becoming problem solvers and reflective writers in mathematics. In the end I questioned the students to see how they had changed since September. Students had become confident and independent writers, and they understood what it meant to be a problem solver. Two students wrote:

I can think of more answers. (mathematics journal, Dec. 3, Mitchel)

I've learned how to do harder problems. (mathematics journal, Dec. 3, Sam)

I now believe writing belongs in mathematics and is as important in developing students' mathematical knowledge as numbers and computation. It was my student-to-teacher interactions and my open-ended questions that guided students to write reflectively. I will always have a classroom of diverse learners and I now feel confident I can meet individual needs of students and lead them in their learning. Mathematics journals will guide my teaching.

References

Brewer, R. (1994-98). *Exemplars*. Underhill, Vermont: Exemplars.

Countryman, J. (1992). *Writing to Learn Mathematics*. Portsmouth, NH: Heinemann Education Books, Inc.

Introducing Rubrics to Students

By Deb Armitage, Exemplars Consultant




A rubric is an assessment guide that reflects content standards and performance standards. Rubrics describe the features expected for student work to receive each of the levels/scores on the chosen scale. An assessment rubric tells us what is important, defines what work meets a standard, and allows us to distinguish between different levels of performance.

Students need to understand the assessment guide that is being used to assess their performance. Teachers often begin this understanding by developing rubrics with students that do not address a specific content area. Together, they develop rubrics around classroom management, playground behavior, homework, lunchroom behavior, following criteria with a substitute teacher, etc. Developing rubrics with students around the best chocolate chip cookie, sneaker, crayon, etc. is also an informative activity to help students understand performance levels. After building a number of rubrics with students, a teacher can introduce the Exemplars rubric. Since the students will have an understanding of what an assessment guide is, they will be ready to focus on the criteria and performance levels of the rubric.

The following pages demonstrate a series of rubrics that were developed by teachers to stir your imagination as you decide what assessment guide would be best to begin with your students.

It is very important for your students to develop their “own” first rubric.

Lunchroom Behavior Rubric

Criteria	Level 1 	Level 2 	Level 3 
Waiting In Line	Outside voice Touching, pushing, shoving Frontsies/backsies	Inside voice Occasional holding spots for an individual or cuts in line	Stage whispers (just above a whisper) Joins line at the end as enters the cafeteria
Table Manners	<u>Rude</u> Stealing seat Eating off other's plate Poking/grabbing Throwing food Singing Wandering off	<u>Family</u> Kind words Elbows allowed Using fingers Eating at spot but standing	<u>Restaurant</u> Using please, thank you, excuse me No elbows No singing Chews with mouth closed Using utensils Staying in seat
Noise Level	Outside voice	Inside voice	Stage whispers
Dismissal Prep	Dirty table/floor No recycling	Mostly clean table Mostly clean floor Some recycling	Clean table Clean floor Correct recycling

Student Set-Up

	1 Exhibiting <u>none</u>	2 Exhibiting <u>one</u>	3 Exhibiting <u>two or more</u>
Arrival	On Time Entering quietly Walking Sitting immediately	On Time Entering quietly Walking Sitting immediately	On Time Entering quietly Walking Sitting immediately
Arrival Attitude	Pleasant greeting Quiet voice Getting ready to start the school day	Pleasant greeting Quiet voice Getting ready to start the school day	Pleasant greeting Quiet voice Getting ready to start the school day
Homework/ Materials	Folder with homework completed Homework in good condition	Folder with homework completed Homework in good condition	Folder with homework completed Homework in good condition

Chocolate Chip Cookie Scoring Guide

Criteria	Level 1	Level 2	Level 3	Level 4
Size	Minuscule less than 2 inches across	Medium more than 2 inches but less than three inches	Large more than 3 inches across	
Number of Chips	No chips evident	A few chips less than 10	Enough chips 11-20 chips	Lots of chips more than 21
Texture	Hard stale	Crispy dry	Firm circumference with soft center	Soft moist
Taste	No Flavor Evident	A Little Flavor	Enough Flavor	Lots of Flavor Um, um, good

Cookie Trials

Cookie A

Criteria	Level Score
Size	
Number of Chips	
Texture	
Taste	

Cookie B

Criteria	Level Score
Size	
Number of Chips	
Texture	
Taste	



Cookie C

Criteria	Level Score
Size	
Number of Chips	
Texture	
Taste	

Cookie D

Criteria	Level Score
Size	
Number of Chips	
Texture	
Taste	

Bathroom Rubric

	Got It 	Keep Practicing 
Knock First		
Quiet and Respectful Use		
Four Squares Only		
Quick as You Can		
Flush When Finished		
Doors Unlocked		
Hands Washed!		

Exemplars® Classic 3-Level Rubric

Level	Understanding	Strategies, Reasoning, Procedures	Communication
Novice	<ul style="list-style-type: none"> There is no solution, or the solution has no relationship to the task. Inappropriate concepts are applied and/or procedures are used. The solution addresses none of the mathematical components presented in the task. 	<ul style="list-style-type: none"> No evidence of a strategy or procedure, or uses a strategy that does not help solve the problem. No evidence of mathematical reasoning. There were so many errors in mathematical procedures that the problem could not be solved. 	<ul style="list-style-type: none"> There is no explanation of the solution, the explanation can not be understood or it is unrelated to the problem. There is no use or inappropriate use of mathematical representations (e.g. figures diagrams, graphs, tables, etc.).
Apprentice	<ul style="list-style-type: none"> The solution is not complete indicating that parts of the problem are not understood. The solution addresses some, but not all of the mathematical components presented in the task. 	<ul style="list-style-type: none"> Uses a strategy that is partially useful, leading some way toward a solution, but not to a full solution of the problem. Some evidence of mathematical reasoning. Could not completely carry out mathematical procedures. Some parts may be correct, but a correct answer is not achieved. 	<ul style="list-style-type: none"> There is no use, or mostly inappropriate use, of mathematical terminology and notation. There is an incomplete explanation; it may not be clearly presented. There is some use of appropriate mathematical representation.
Practitioner	<ul style="list-style-type: none"> The solution shows that the student has a broad understanding of the problem and the major concepts necessary for its solution. The solution addresses all of the mathematical components presented in the task. 	<ul style="list-style-type: none"> Uses a strategy that leads to a solution of the problem. Uses effective mathematical reasoning. Mathematical procedures used. All parts are correct and a correct answer is achieved. 	<ul style="list-style-type: none"> There is some use of mathematical terminology and notation appropriate of the problem. There is a clear explanation. There is appropriate use of accurate mathematical representation. There is effective use of mathematical terminology and notation.
Expert	<ul style="list-style-type: none"> The solution shows a deep understanding of the problem including the ability to identify the appropriate mathematical concepts and the information necessary for its solution. The solution completely addresses all mathematical components presented in the task. The solution puts to use the underlying mathematical concepts upon which the task is designed. 	<ul style="list-style-type: none"> Uses a very efficient and sophisticated strategy leading directly to a solution. Employs refined and complex reasoning. Applies procedures accurately to correctly solve the problem and verify the results. Verifies solution and/or evaluates the reasonableness of the solution. Makes mathematically relevant observations and/or connections. 	<ul style="list-style-type: none"> There is a clear, effective explanation detailing how the problem is solved. All of the steps are included so that the reader does not need to infer how and why decisions were made. Mathematical representing is actively used as a means of communicating ideas related to the solution of the problem. There is precise and appropriate use of mathematical terminology and notation.

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Exemplars® Classic 5-Level Rubric*

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Novice	No strategy is chosen, or a strategy is chosen that will not lead to a solution. Little or no evidence of engagement in the task present.	Arguments are made with no mathematical basis. No correct reasoning nor justification for reasoning is present.	No awareness of audience or purpose is communicated. Little or no communication of an approach is evident Everyday, familiar language is used to communicate ideas.	No connections are made.	No attempt is made to construct mathematical representations.
Apprentice	A partially correct strategy is chosen, or a correct strategy for only solving part of the task is chosen. Evidence of drawing on some previous knowledge is present, showing some relevant engagement in the task.	Arguments are made with some mathematical basis. Some correct reasoning or justification for reasoning is present with trial and error, or unsystematic trying of several cases.	Some awareness of audience or purpose is communicated, and may take place in the form of paraphrasing of the task. Some communication of an approach is evident through verbal/written accounts and explanations, use of diagrams or objects, writing, and using mathematical symbols. Some formal math language is used, and examples are provided to communicate ideas.	Some attempt to relate the task to other subjects or to own interests and experiences is made.	An attempt is made to construct mathematical representations to record and communicate problem solving.

*Based on revised NCTM standards.

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Exemplars® Classic 5-Level Rubric cont.

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Practitioner	<p>A correct strategy is chosen based on mathematical situation in the task.</p> <p>Planning or monitoring of strategy is evident.</p> <p>Evidence of solidifying prior knowledge and applying it to the problem solving situation is present.</p> <p>Note: The practitioner must achieve a correct answer.</p>	<p>Arguments are constructed with adequate mathematical basis.</p> <p>A systematic approach and / or justification of correct reasoning is present. This may lead to...</p> <ul style="list-style-type: none"> clarification of the task. exploration of mathematical phenomenon. noting patterns, structures and regularities. 	<p>A sense of audience or purpose is communicated. and / or</p> <p>Communication of an approach is evident through a methodical, organized, coherent sequenced and labeled response.</p> <p>Formal math language is used throughout the solution to share and clarify ideas.</p>	<p>Mathematical connections or observations are recognized.</p>	<p>Appropriate and accurate mathematical representations are constructed and refined to solve problems or portray solutions.</p>
Expert	<p>An efficient strategy is chosen and progress towards a solution is evaluated.</p> <p>Adjustments in strategy, if necessary, are made along the way, and / or alternative strategies are considered.</p> <p>Evidence of analyzing the situation in mathematical terms, and extending prior knowledge is present.</p> <p>Note: The expert must achieve a correct answer.</p>	<p>Deductive arguments are used to justify decisions and may result in formal proofs.</p> <p>Evidence is used to justify and support decisions made and conclusions reached. This may lead to...</p> <ul style="list-style-type: none"> testing and accepting or rejecting of a hypothesis or conjecture. explanation of phenomenon. generalizing and extending the solution to other cases. 	<p>A sense of audience and purpose is communicated. and / or</p> <p>Communication at the Practitioner level is achieved, and communication of argument is supported by mathematical properties.</p> <p>Precise math language and symbolic notation are used to consolidate math thinking and to communicate ideas.</p>	<p>Mathematical connections or observations are used to extend the solution.</p>	<p>Abstract or symbolic mathematical representations are constructed to analyze relationships, extend thinking, and clarify or interpret phenomenon.</p>

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Exemplars NCTM Classic 5-Level Rubric

The Exemplars Rubric is based on the following NCTM Standards:

Problem Solving

Instructional programs from pre-kindergarten through grade 12 should enable all students to:

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof

Instructional programs from pre-kindergarten through grade 12 should enable all students to:

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to:

- Organize and consolidate their mathematical thinking through communication
- Communicate mathematical thinking coherently and clearly to peers, teachers and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to:

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Representation

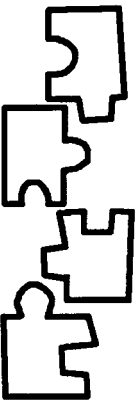
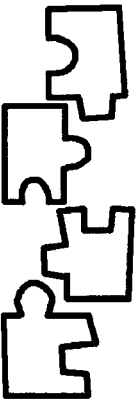
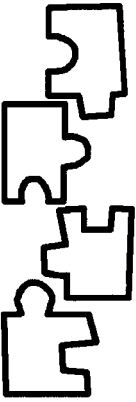
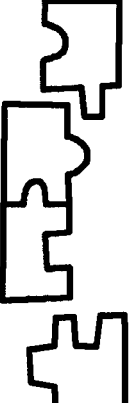
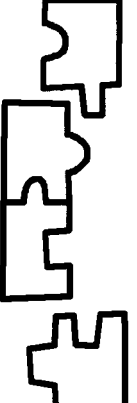
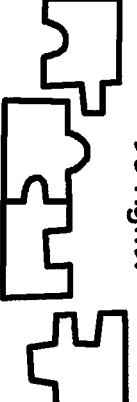
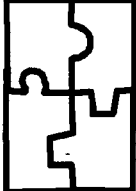
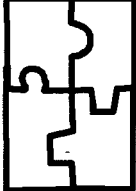
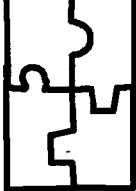
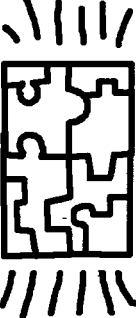
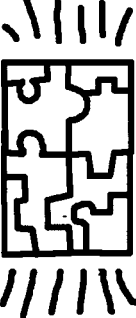
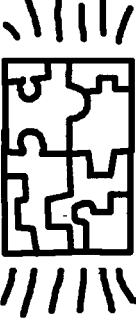
Instructional programs from pre-kindergarten through grade 12 should enable all students to:

- Create and use representations to organize, record and communicate mathematical ideas
- Select, apply and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social and mathematical phenomenon

NCTM Standard Rubric Glossary of Terms:






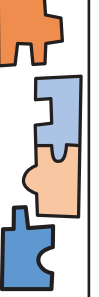
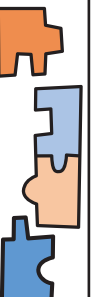
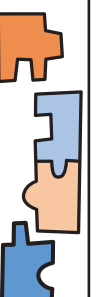
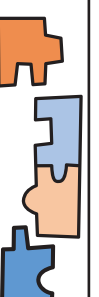
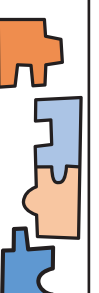










- Analyzing:** Separating an abstract entity into its constituent elements, determining essential features, examining carefully and in detail
- Conjecture:** Formulating a theory without proof
- Deductive:** The process in which a conclusion follows necessarily from the premises or facts presented. Moves from general to particular.
- Efficient:** Produced with least waste of time and effort
- Engagement:** Occupying attention or efforts
- Evaluated:** Quality judged
- Hypothesis:** An assumed proposition
- Justification:** A reason, fact or circumstance that is shown or proven right or reasonable.
- Phenomenon:** A fact, occurrence or circumstance
- Systematic:** Methodical, planned or ordered

Exemplars® Classic Primary Math Rubric

Level	Understanding	Strategy, Reasoning, Procedure	Communication
Novice Makes an Effort No understanding	I don't understand the problem. 	I'm not sure what steps to take. 	I'm not sure. I can't explain it. 
Apprentice Ok, good try Unclear if student understands	I think I understand. 	Got started. I'm still thinking. 	I can explain some of what I did. My answer may be right. 
Practitioner Very good, clear Strong understanding	I got it. 	I have a plan. I'm not sure of the steps. 	I can tell and show you. I can explain. 
Expert Wow, awesome! Excellent understanding	I got it. I can prove it. I can do other problems like this. 	I can show more than one plan. I can prove it. 	I clearly showed & explained the problem with details. 

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Exemplars® Standard Primary Math Rubric

Level	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Novice Makes an effort. No or little understanding.	I did not understand the problem. 	My math thinking is not correct. 	I used no math language and/or math notation. 	I did not notice anything about the problem or the numbers in my work. 	I did not use a math representation to help solve the problem and explain my work. 
Apprentice Okay, good try. Unclear if student understands.	I understand only part of the problem. My strategy works for part of the problem. 	Some of my math thinking is correct. 	I used some math language and/or math notation. 	I tried to notice something, but it is not about the math in the problem. 	I tried to use a math representation to help solve the problem and explain my work, but it has mistakes in it. 
Practitioner Excellent. Clear. Strong understanding. Meets the standard.	I understand the problem and my strategy works. My answer is correct. 	All of my math thinking is correct. 	I used math language and/or math notation accurately throughout my work. 	I noticed something about my math work. 	I made a math representation to help solve the problem and explain my work, and it is labeled and correct. 
Expert Wow, awesome! Exceptional understanding!	I understand the problem. My answer is correct. I used a rule, and/or verified that my strategy is correct. 	I showed that I knew more about a math idea that I used in my plan. Or, I explained my rule. 	I used a lot of specific math language and/or notation accurately throughout my work. 	I noticed something in my work, and used that to extend my answer and/or I showed how this problem is like another problem. 	I used another math representation to help solve the problem and explain my work in another way. 

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Exemplars® Classic Rubric for Student Self Evaluation

Novice Apprentice Practitioner Expert

Understanding

I did not understand the problem.

I understood parts of the problem. I got started, but I couldn't finish.

I got it. I understood the problem and have an appropriate solution. All parts of the problem are addressed.

I got it!! I did it in new ways and showed you how it worked. I can tell you what math concepts are used.

Strategies, Reasoning, Procedures

I couldn't get started. I don't know how to begin.

I am stuck. I have part of the solution, but now I don't know what to do. I'm not sure my answer is right. I could use some help.

I have a correct solution. I used a plan to solve the problem.

My solution is effective and inventive. I used big math ideas to solve the problem. I addressed the important details. I showed you some other ways I can solve this problem. I checked to make sure my answer was right.

Communication

I did not explain how I solved the problem. I didn't use pictures, tables, or graphs to show you how I solved the problem.

I explained some of what I did. I tried to use pictures, tables, graphs and numbers to explain how I did the problem.

I clearly explained how I solved the problem. I used math language and pictures, tables, graphs and numbers to explain how I did the problem.

I clearly detailed how I solved the problem. I included all the steps so you don't have to guess what I did. I used words, numbers, pictures, graphs and/or models.

Sample Task

A Puzzle

Marc, Amanda and Jacob were putting a 100-piece jigsaw puzzle together. Marc placed $\frac{1}{4}$ of the pieces in the puzzle. Amanda placed $\frac{1}{5}$ of the pieces into the puzzle. Jacob placed the remaining pieces in the puzzle and told Marc and Amanda that he knew exactly how many pieces he had put in the puzzle. How many pieces did Jacob put in the jigsaw puzzle? Show all your math thinking.

Sample Task cont.

A Puzzle

Suggested Grade Span

Grades 3–5

Grade(s) in Which Task was Piloted

Grade 5

Task

Marc, Amanda and Jacob were putting a 100-piece jigsaw puzzle together. Marc placed $\frac{1}{4}$ of the pieces in the puzzle. Amanda placed $\frac{1}{5}$ of the pieces into the puzzle. Jacob placed the remaining pieces in the puzzle and told Marc and Amanda that he knew exactly how many pieces he had put in the puzzle. How many pieces did Jacob put in the jigsaw puzzle? Show all your math thinking.

Alternative Versions of Task

More Accessible Version:

Marc, Amanda and Jacob were putting a 100 piece jigsaw puzzle together. Marc placed $\frac{1}{2}$ of the pieces in the puzzle. Amanda placed $\frac{1}{4}$ of the pieces into the puzzle. Jacob placed the remaining pieces in the puzzle and told Marc and Amanda that he knew exactly how many pieces he had put in the puzzle. How many pieces did Jacob put in the jigsaw puzzle? Show all your math thinking.

More Challenging Version:

Marc, Amanda and Jacob were putting a jigsaw puzzle together. Marc placed $\frac{1}{4}$ of the pieces in the puzzle. Amanda placed $\frac{1}{5}$ of the pieces into the puzzle. Jacob placed 55 pieces in the puzzle. How many pieces are in the jigsaw puzzle? Show all your math thinking.

NCTM Content Standards and Evidence

Number and Operations Standard for Grades 3–5

Instructional programs from pre-kindergarten through grade 12 should enable students to —

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
 - **NCTM Evidence:** Develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers
 - **Exemplars Task-Specific Evidence:** This task requires students to understand that $\frac{1}{4}$ of a set of a collection of 100 things is 25 and that $\frac{1}{5}$ of a set of a collection of 100 things is 20.

Time/Context/Qualifiers/Tip(s) From Piloting Teacher

This is a short- to medium-length task. Students should be aware of fractions as part of a collection.

Sample Task cont.

Links

This task can be linked with a free time activity of working on a crossword puzzle. This task also links with *Fraction Action* by L. Leedy, *Each Orange Had 8 Slices* by P. Giganti, Jr. or *Eating Fractions* by B. McMillan.

Common Strategies Used to Solve This Task

This was a challenging task for most fifth graders. Most started by finding the number of puzzle pieces placed by Marc and Amanda. They then subtracted that number from 100 to find the number of pieces that Jacob placed. Some kids tried adding the fractions and coming up with Jacob putting $11/20$ of the puzzle together. They then had to find $11/20$ of the 100 pieces.

Possible Solutions

Original Version:

Marc placed $1/4$ of 100 = 25 pieces.

Amanda placed $1/5$ of 100 = 20 pieces.

That left Jacob (who placed $11/20$ of the pieces) placing $(100 - 45)$ 55 pieces.

More Accessible Version:

Marc placed $1/2$ of 100 = 50 pieces.

Amanda placed $1/4$ of 100 = 25 pieces.

That left Jacob (who placed $1/4$ of the pieces) placing $(100 - 75)$ 25 pieces.

More Challenging Version:

Marc and Amanda placed $(1/4 + 1/5)$ $9/20$ of the pieces of the puzzle which means Jacob placed $(20/20 - 9/20)$ $11/20$ of the pieces of the puzzle which is equal to 55 pieces. Dividing by 11 gives you $1/20$ of the puzzle equal to 5 pieces. Then $20/20$ of the puzzle or the whole puzzle is equal to (5×20) 100 pieces.

Task-Specific Assessment Notes

General Notes

Students should be allowed to use manipulatives if they feel they may help.

Novice

The Novice will not have a strategy that will lead to a solution. They will not be able to relate $1/4$ of the 100 pieces to 25 pieces. They will not have a working understanding of representing fractions as parts of a collection.

Apprentice

The Apprentice will have some working knowledge of fractions and will be able to engage in part of the problem. They may be able to find that Jacob placed $11/20$ th of the puzzle but not be able to compare that to the number of pieces.

Sample Task cont.

Practitioner

The Practitioner will be able to find a correct solution. They will show a good understanding of fractions as part of a collection. A sense of audience or purpose is communicated and a systematic approach and/or justification of correct reasoning is present.

Expert

The Expert will have all the Practitioner has and more. They may make a strong connection between decimals, fractions and percents. They may make a connection between this problem and other problems they have solved or a problem in the real world.

Novice

You got to make a puzzle that is 100 pieces.

100 = 1 puzzle

student	pieces
marc	$\frac{1}{4}$
Amanda	$\frac{1}{5}$
Jacob	$\frac{1}{9}$

$$\frac{1}{4} + \frac{1}{5} = \frac{2}{9}$$

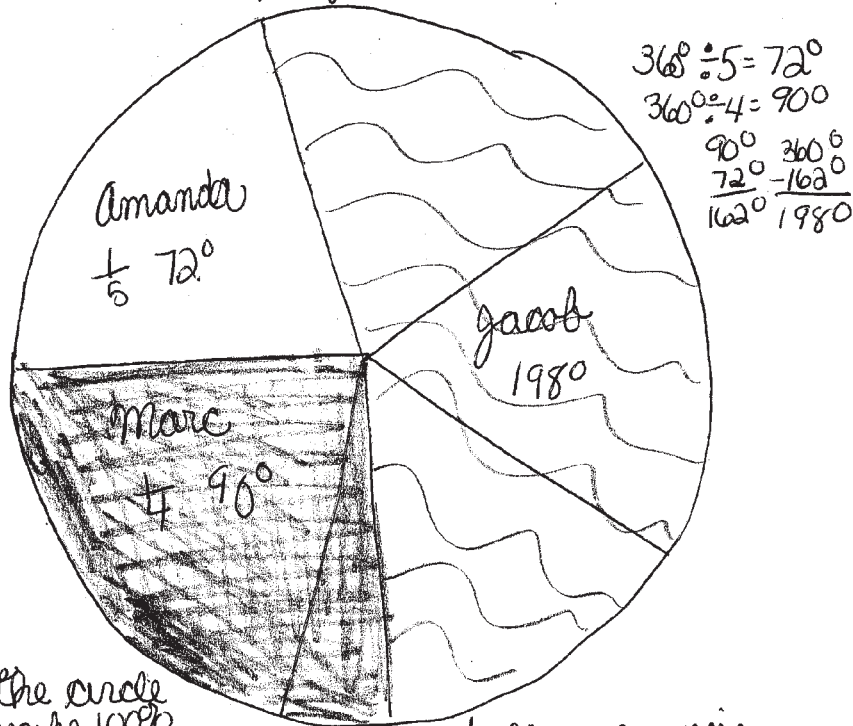
marc put $\frac{1}{4}$ pieces in.
 Amanda put $\frac{1}{5}$ pieces in.
 Jacob put $\frac{1}{9}$ pieces in.

Adding $\frac{1}{4} + \frac{1}{5} = \frac{2}{9}$ is not a strategy that will solve the problem.

The student could not find the number of pieces in a fraction of the puzzle.

Apprentice

I will make a pie graph as my reasoning.



The circle can be 100% so $\frac{1}{4}$ of 100% is 25 pieces and $\frac{1}{5}$ of 100 is 20 pieces.

Knowing that $\frac{1}{4}$ of the puzzle is 25 pieces and $\frac{1}{5}$ of the puzzle is 20 pieces solves part of the problem.

The student does not have a strategy to find the number of remaining pieces.

The circle graph is accurate but did not help the student solve the problem.

Practitioner

How many puzzle pieces did Jacob put in the puzzle? I'm good with percents so that is what I'll do to solve the question.

THE PUZZLE

KID	FRACTION	PERCENT	PIECES
MARC	$\frac{1}{4}$	25%	25
AMANDA	$\frac{1}{5}$	20%	20
JACOB	$\frac{11}{20}$	55%	(55)

my work

$$\frac{1}{4}(100) = 25\%$$

$$\frac{1}{5}(100) = 20\%$$

$$\frac{1}{4} + \frac{1}{5} = \frac{5}{20} + \frac{4}{20} = \frac{9}{20}$$

$$25 + 20 = 45\%$$

$$45 + 55 = 100$$

$$\frac{20}{20} - \frac{9}{20} = \frac{11}{20}$$

I noticed that Jacob put almost $\frac{1}{2}$ of the puzzle together.

A correct solution is achieved.

The student's work communicates the student's strategy and mathematical reasoning.

The observation is correct.

Expert

I have to find how many pieces Jacob put in the puzzle. My plan is to make a table and do fractions and percents.

Name	Marc	Amanda	Jacob
Fraction of pieces put in puzzle	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{11}{20}$
Number of pieces	25	20	55
Percents	25%	20%	55%

$$\frac{1}{4} = \frac{5}{20}$$

$$\frac{1}{5} = \frac{4}{20}$$

$$\frac{9}{20}$$

$$\frac{20}{20}$$

$$-\frac{9}{20}$$

$$\frac{11}{20}$$

Jacob put 55 pieces in the puzzle

$$\frac{1}{4} \text{ of } 100 = 25$$

$$\frac{1}{5} \text{ of } 100 = 20$$

$$\begin{array}{r} 25 \\ 20 \\ \hline 45 \\ 100 \\ -45 \\ \hline 55 \end{array}$$

The equations help communicate the student's mathematical reasoning and strategy.

Expert cont.

You can Prove that I am right by using decimals. They are like percents

$$\frac{1}{4} \text{ of } 100 = .25$$

$$\frac{1}{5} \text{ of } 100 = .20$$

$$\begin{array}{r} .25 \\ \times 4 \\ \hline 1.00 \end{array} \quad \begin{array}{r} .25 \\ .20 \\ \hline .45 \end{array}$$

$$\begin{array}{r} .20 \\ \times 5 \\ \hline 1.00 \end{array} \quad \begin{array}{r} 100 \\ - .45 \\ \hline .55 \end{array}$$

$$.55 = 55\% = \frac{55}{100} \quad \text{.55}$$

See, it is right

A correct solution is achieved.

This was like the painting the fence problem. I had to see how many boards 3 kids painted but it was 200 boards and the fractions were $\frac{2}{4}$ and $\frac{3}{5}$. I did Fractions and Percents. I didn't think of decimals on that one.

The student solved with fractions and decimals to verify the solution.

The student makes a connection to a similar problem.

Exemplars® Standard Rubric Scoring Notes

Please copy and distribute these for use amongst your teachers.

Exemplars Level	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Novice					
Apprentice					
Practitioner					
Expert					

Comments:

Grade Level

Student's Name: _____

Date: _____

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Task Organizer*

Title of Task: _____

Briefly restate the gist of the question: _____

Similar Tasks

Math tools that may help:

- ruler
- graph paper
- protractor
- compass
- calculator
- manipulatives
- spread sheet
- internet
- other

Strategies that may work:

- draw a picture
- work backwards
- identify a pattern
- act it out
- solve a simpler version
- make a table
- take a survey

Connection Ideas

My estimate of the solution:

Underlying mathematics in the task:

Relevant math language, symbols, and notation:

Units to Label My Solution:

Formulas that may be appropriate to use:

Math representations that would be appropriate to use to solve the problem or communicate the solution:

- | | | | |
|---------------------------------------|---|--|-------------------------------------|
| <input type="checkbox"/> Chart | <input type="checkbox"/> Table | <input type="checkbox"/> Bar Graph | <input type="checkbox"/> Line Graph |
| <input type="checkbox"/> Pictograph | <input type="checkbox"/> Diagram | <input type="checkbox"/> Model | <input type="checkbox"/> Pie Graph |
| <input type="checkbox"/> Scatter Plot | <input type="checkbox"/> Line Plot | <input type="checkbox"/> Box and Whiskers Plot | |
| | <input type="checkbox"/> Stem and Leaf Plot | | |

*Developed by Carol McNair.

*Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

Title of Task _____ Standard(s) Addressed _____

GE(s) Addressed _____ Program Link _____

Underlying Mathematical
Concepts

Problem Solving
Strategies/Representation

Mathematical Language

Possible Solution(s)

Connections

Related Tasks

What I Know	What I Want to Know	What I Learned