

Grade 6 Sample Instructional Math Task

Engagement Image to Launch Task

Teachers use this resource to pique student curiosity.



Grade 6 Sample Instructional Math Task

How Big is the Property

Hectar is in dispute over the amount of property taxes he owes to the state for his farmland. Property taxes on farmland are set by the state government and are determined by the total area of the property, measured in acres. The state is using the aerial photo below to define the total size of the property.

The boundaries for the property can be described using coordinates on a grid overlaid on the aerial photo. If Hectar walks the property boundary, he arrives at coordinates $(320, 0)$, $(700, 0)$, $(700, 340)$, $(320, 340)$, $(320, 520)$, $(0, 520)$, $(0, 180)$, $(320, 180)$, $(320, 0)$. All coordinates are defined in units of feet.

The state has a property tax rate of \$326 per acre. An acre is equal to 43,560 square feet. As a result, the town is currently charging Hectar \$2,444 in property taxes. Hectar believes he is paying too much in property taxes.

Help Hectar prepare a letter to the state government explaining how much he believes he should be paying the state for his property. Be sure to include all of your mathematical thinking.

How Big is the Property?



Instructional

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Alternative Versions of the Task

More Accessible Version

Hectar is in dispute over the amount of property taxes he owes to the state for his farmland. Property taxes on farmland are set by the state government and are determined by the total area of the property, measured in acres. The state is using the aerial photo below to define the total size of the property.

The boundaries for the property can be described using coordinates on a grid overlaid on the aerial photo. If Hectar walks the property boundary, he arrives at coordinates, (300, 0), (700, 0), (700, 300), (300, 300), (300, 500), (0, 500), (0, 200), (300, 200), (300, 0). All coordinates are defined in units of feet.

The state has a property tax rate of \$300 per acre. An acre is equal to 43,560 square feet. As a result, the town is currently charging Hectar \$2,500 in property taxes. Hectar believes he is paying too much in property taxes.

Help Hectar prepare a letter to the state government explaining how much he believes he should be paying the state for his property. Be sure to include your mathematical thinking.

More Challenging Version

Hectar is in dispute over the amount of property taxes he owes to the state for his farmland. Property taxes on farmland are set by the state government and are determined by the total area of the property, measured in acres. The state is using the aerial photo below to define the total size of the property.

The boundaries for the property can be described using coordinates on a grid overlaid on the aerial photo. If Hectar walks the property boundary, he arrives at coordinates (320, 0), (700, 0), (700, 340), (320, 520), (0, 520), (0, 180), and back to (320, 0). All coordinates are defined in units of feet.

The state has a property tax rate of \$326 per acre. An acre is equal to 43,560 square feet. As a result, the town is currently charging Hectar \$2,444 in property taxes. Hectar believes he is paying too much in property taxes.

Help Hectar prepare a letter to the state government explaining how much he believes he should be paying the state for his property. Be sure to include your mathematical thinking.

How Big is the Property?



Planning Sheet

How Big is the Property?

Common Core Task Alignments

Mathematical Practices: MP.2 MP.3 MP.5 MP.6

Grade 6 Content Standards: 6.G.A.3

Common Core Standards and Evidence

6.G.A.3

Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Exemplars Task-Specific Evidence

This task requires students to determine the area of farmland based on coordinate points and convert square feet to acres. They will use this information to determine how much property tax the owner should be paying and write a letter making a case for the change in cost.

Underlying Mathematical Concepts

- Distance on the coordinate plane
- Area
- Composite figures
- Application of unit rates
- Scale

Possible Problem-Solving Strategies

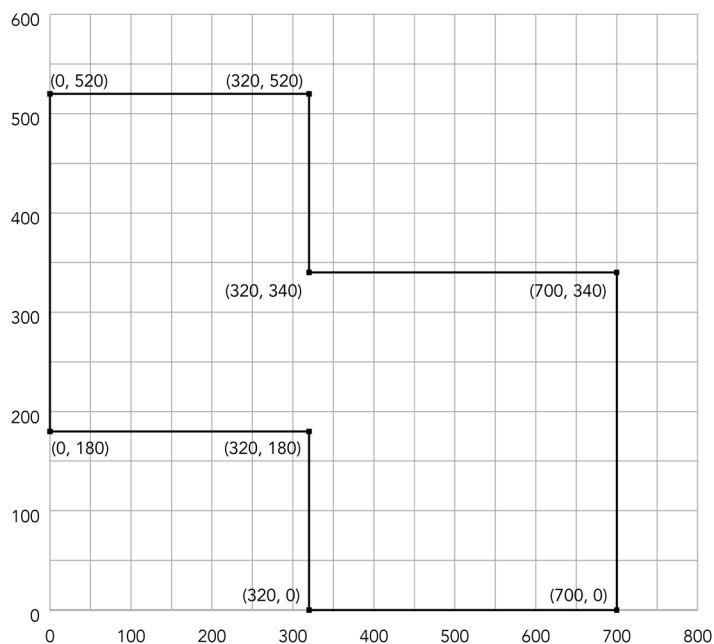
- Graph points on a coordinate plane
- Create a scale drawing
- Table
- Calculate distance using coordinates

Possible Mathematical Vocabulary/Symbolic Representation

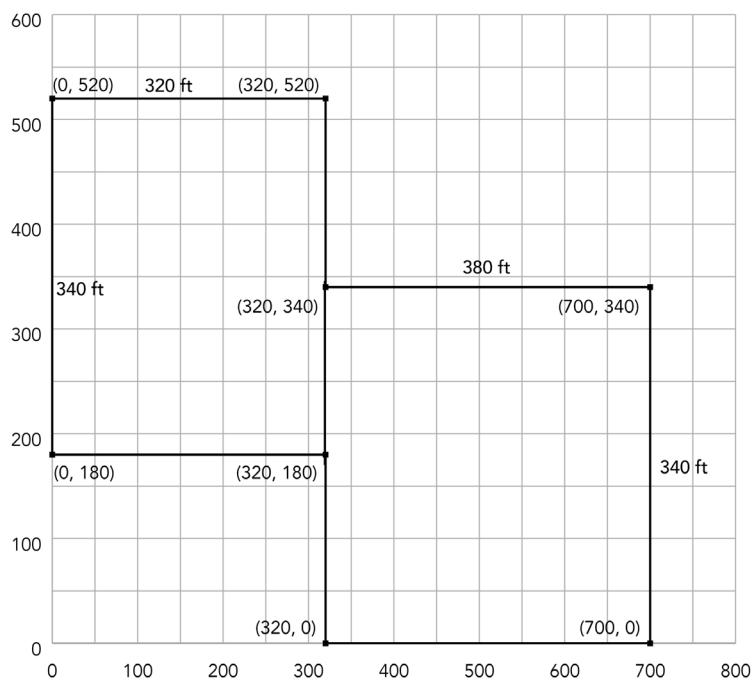
- Coordinates
- Coordinate plane
- Distance
- Area
- Vertices
- sq. ft (ft²)
- Acres
- Quadrant
- Total/Sum
- Length
- Grid
- x-axis/y-axis

Possible Solutions

Hectar should be paying \$1,793 in property taxes. The area of his property is about 5.5 acres. The boundaries of Hectar's property are shown on the coordinate grid:



Students can find the area of Hectar's property by decomposing the property into 2 rectangles as shown and use the coordinates to find the dimensions and area of each rectangle.



The rectangle on the left is 340 ft ($520 - 180 = 340$) by 320 ft ($320 - 0 = 320$) so its area is 108,800 sq ft.

The rectangle on the right is 340 ft ($340 - 0 = 340$) by 380 ft ($700 - 320 = 380$) so its area is 129,200 sq ft.

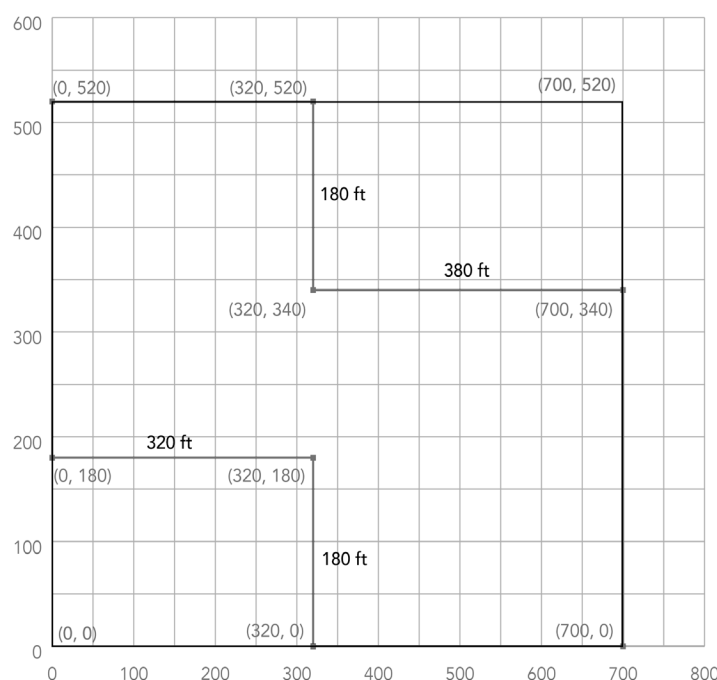
Total area = $108,800 + 129,200 = 238,000$ sq ft

Total Area $\div$ sq ft per acre $\approx$ Total acres

$238,000$ sq ft $\div$ $43,560$ sq ft per acre ≈ 5.5 acres

Cost: 5.5 acres $\times$ $\$326$ per acre = $\$1,793$

Students could also find the area of the property by first finding the area of the rectangle that would surround the boundaries of the property. The extra area that isn't part of the property would then be subtracted from that area.



The dimensions of the larger rectangle are 520 ft by 700 ft

Area of the green figure: $520 \times 700 = 364,000$ sq ft

The dimensions of the smaller extra rectangle are:

320 ft ($320 - 0 = 320$) by 180 ft ($180 - 0 = 180$)

Area of the smaller extra rectangle: $320 \times 180 = 57,600$ sq ft

The dimensions of the larger extra rectangle are:

380 ft ($700 - 320 = 380$) by 180 ft ($520 - 340 = 180$)

Area of the larger extra rectangle: $380 \times 180 = 68,400$ sq ft

Area of the property:

$364,000$ sq. ft $- 57,600$ sq. ft $- 68,400$ sq. ft = $238,000$ sq ft

Or: $364,000$ sq. ft $- (57,600$ sq. ft $+ 68,400$ sq. ft) = $238,000$ sq ft

$238,000$ sq ft $\div$ $43,560$ sq ft per acre ≈ 5.5 acres

Cost: 5.5 acres $\times$ $\$326$ per acre = $\$1,793$

Grade 6 Sample Summative Assessment Math Task

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Teachers use this resource to pique student curiosity.



Summative

Grade 6 Sample Summative Assessment Math Task

Herding Cats

Maru owns a free-roam cat rescue. Maru is working to enclose her roaming area with a cat-safe electric fence. Maru knows the cost of an electric fence installed is \$0.89 per foot. The Meow Safe Fencing Company has provided an estimated price of \$3,190 for fencing in the total property.

The boundaries for the property can be described using coordinates on a scaled grid overlaid on an aerial photo. Each unit is 30 feet. The boundary coordinates are (1, 0), (25, 0), (25, 22), (21, 22), (21, 16), (5, 16), (5, 22), and (1, 22).

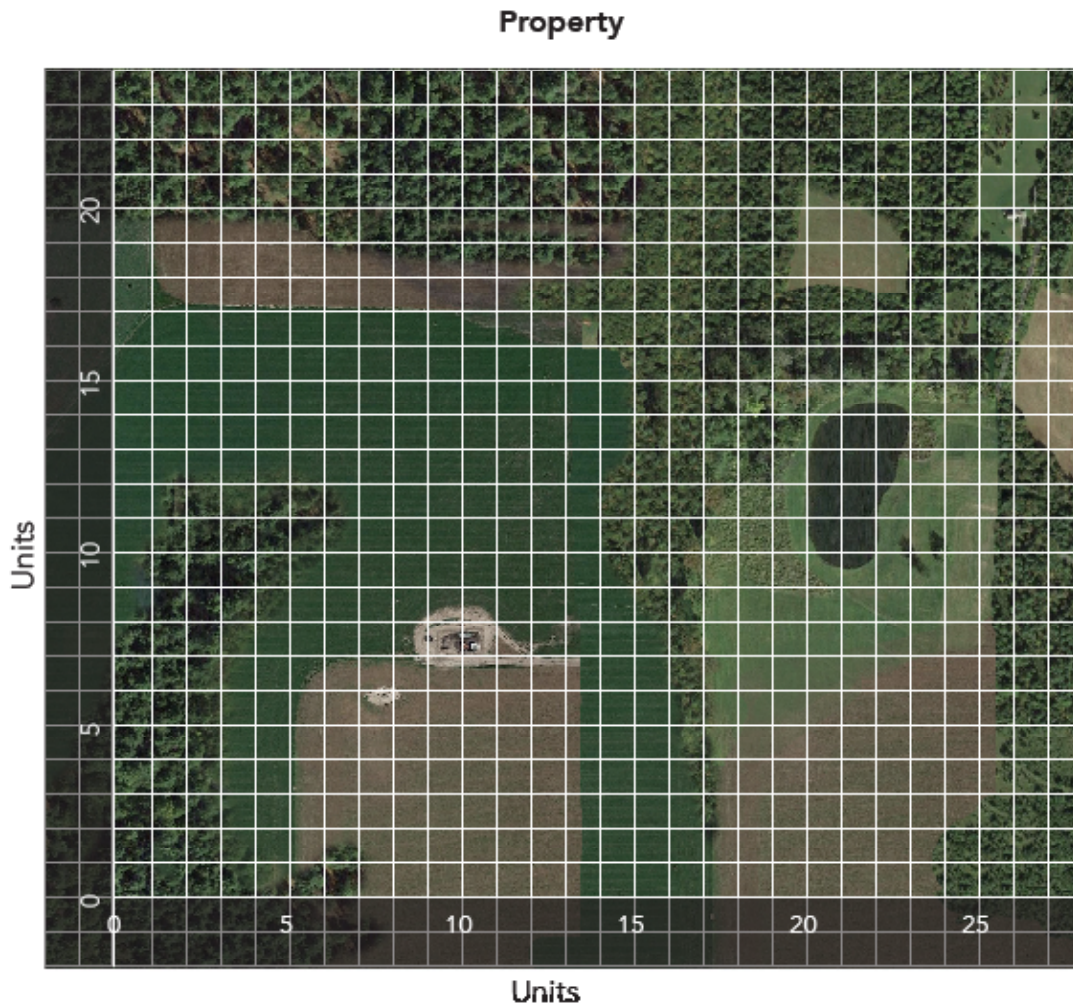
Write a letter to the Meow Safe Fencing Company either accepting or rejecting their offer. Be sure to include all of your mathematical thinking.

Herding Cats

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Write a letter to the Meow Safe Fencing Company either accepting or rejecting their offer. Be sure to include all of your mathematical thinking.



Planning Sheet

Herding Cats

Common Core Task Alignments

Mathematical Practices: MP.2 MP.3 MP.5 MP.6

Grade 6 Content Standards: 6.G.A.3

Common Core Standards and Evidence

6.G.A.3

Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Exemplars Task-Specific Evidence

This task requires students to determine how much it will cost to fence in an area based on a set of coordinate points and given scale. They will also need to write a letter to the fencing company to accept or reject the price the company plans on charging based on their findings.

Underlying Mathematical Concepts

- Graphing points on a coordinate grid
- Distance on the coordinate plane
- Perimeter
- Scaling
- Scale units
- Addition
- Rate

Possible Problem-Solving Strategies

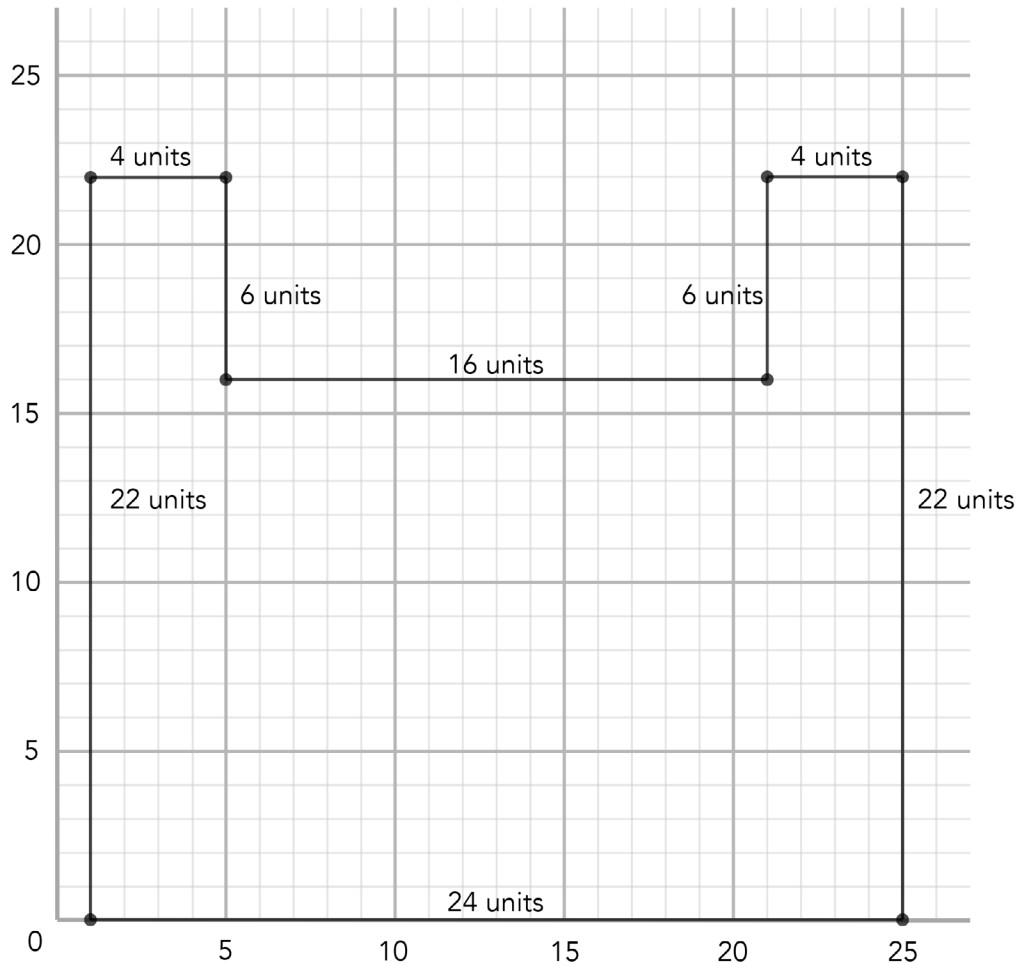
- Graph coordinate points
- Create a scale drawing
- Table
- Calculate distance using coordinates
- Determine cost based on unit cost (rate)

Possible Mathematical Vocabulary/Symbolic Representation

- Coordinates
- Coordinate plane
- Perimeter
- Distance
- Vertices
- Scale
- Axis
- Grid
- Quadrant
- Total/Sum
- Length
- Rate

Possible Solutions

The cost of having the electric fence installed should cost \$2,766.80, which is below what the Meow Safe Fencing Company has offered us. Students may decide The Meow Safe Fencing Company's estimate is reasonable or not and should make an argument based on the information they consider. For example, other additional costs could be factored into their argument.



Line Segment	Length (units)	Actual Length (ft) (unit length x 30)	Cost (actual length x 0.89)
(1, 0) to (25, 0)	24	720	\$640.80
(25, 0) to (25, 22)	22	660	\$587.40
(25, 22) to (21, 22)	4	120	\$106.80
(21, 22) to (21, 16)	6	180	\$160.20
(21, 16) to (5, 16)	16	480	\$427.20
(5, 16) to (5, 22)	6	180	\$160.20
(5, 22) to (1, 22)	4	120	\$106.80
(1, 22) to (1, 0)	22	660	\$587.40
Total	104	3120	\$2,776.80

Possible Connections

- The Meow Safe Fencing Company's estimate is \$423.20 more than the expected cost.
- Determine the area of the roaming space (432 sq units or 388,800 sq ft).
- The roaming area has a perimeter of 1,040 yards.
- The Meow Safe Fencing Company charges \$2.67 per yard of electric fencing.
- Maru could maximize the area and spend less money on fencing if she made the roaming area a rectangle instead of an irregular shape.
- Relate to a similar task and state a math link.
- Cost for fencing can be stated algebraically: $C = 0.89f$.
- C = total cost of fencing for a property
- f = total feet needed for fencing

Novice Scoring Rationale

Criteria and Performance Level	Rationales
Problem Solving <i>Novice</i>	The student confuses needing to find the perimeter of the enclosed space with their strategy of finding the area of the total yard. The student appears to divide the figure into rectangles and find the areas of these figures. This strategy will never lead to a correct solution for this question.
Reasoning & Proof <i>Novice</i>	The student shows little correct reasoning in attempting to solve the task. No correct reasoning is present in finding the perimeter of the yard. Numerous calculations are present but no reasoning is provided for what is being calculated.
Communication <i>Apprentice</i>	The written explanation given for the work is limited and unclear. Work is difficult to understand and hard to follow. Significant interpretation is required to understand the students strategy by the reader. The use of math language is limited.
Connections <i>Novice</i>	No connections are present or attempted by the student.
Representation <i>Apprentice</i>	Based on the drawing included, the student appears to correctly plot the coordinates given on a coordinate plane to create a diagram of the fenced area. The diagram is not labeled. The student than attempt to break the yard in to rectangle to find the area of the total yard. The student does utilize area models for several calculations, although we are unsure what they are working to find.

Novice

P/S	R/P	Com	Con	Rep	A/Level
N	N	A	N	A	N

First I made the yard. Then I split the yard into a rectangle then I saw how many feet there was. then times if 20 4
By 89¢. and get my answer

$12960 \div 89 = \$38348$

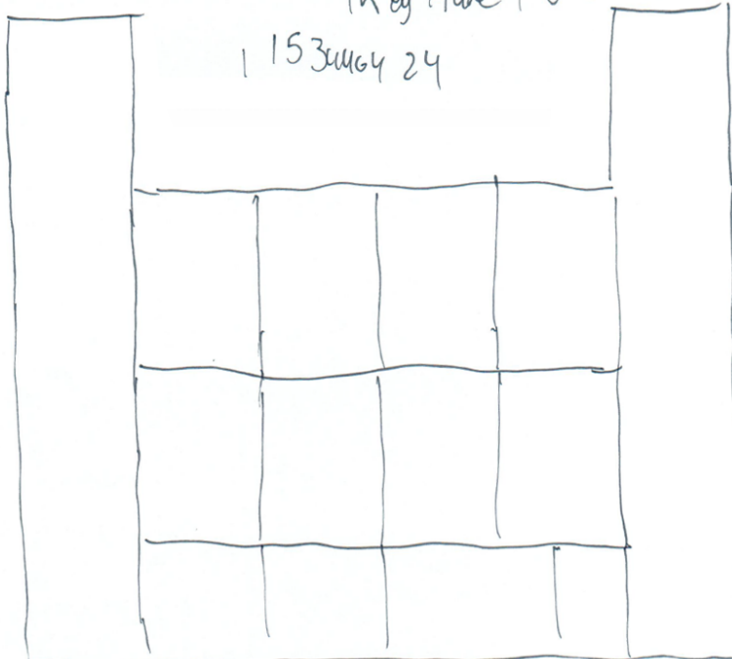
	400	30	2
80	32000	2400	110
9	3600	270	18

32000
3600
2400
270
160
18

140
200
160
+ 32

Answer \$38348

They have 432 feet



	400	30	2
30	12000	900	60

12960

$20 \times 25 =$

432×30

16×4

Apprentice Scoring Rationale

Criteria and Performance Level	Rationales
Problem Solving <i>Apprentice</i>	The student correctly plots and connects the coordinates given to form a diagram of the fenced area. The student incorrectly calculates the perimeter of the polygon in unit lengths (records the bottom segment as 25 units instead of 24 units) to be 105 units instead of 104 units. The student also incorrectly uses 105 units as the perimeter, failing to multiply the measure by 30 feet/unit to get the perimeter in feet. The student multiplies the incorrect perimeter by \$.89 (cost/foot) to get the incorrect cost of \$93.45. These incorrect calculations still leads the student to the correct decision to reject the offer from the fence company.
Reasoning & Proof <i>Apprentice</i>	The student demonstrates some understanding of the underlying concepts of the task, but fails to use the unit rate of 30 feet/unit length to find the perimeter in feet which is needed to determine cost using the unit rate \$.89 cost/foot. The student is inconsistent in finding the unit lengths of all sides of the figure. Errors in computation of the perimeter in unit lengths leads to subsequent errors in the solution process. The calculation $25\text{un}, 22\text{un}, 4\text{un}, 6\text{un}, 16\text{un}, 6\text{un}, 4\text{un}, 22\text{un} = 105 \text{ units} \times 0.89 = 93.45$ is also an incorrect mathematical statement.
Communication <i>Practitioner</i>	The student presents a sequenced response to communicate work. The explanations of the steps to the solution process are clear and easy to follow even though some of the calculations are incorrect. Math vocabulary is limited, yet appropriate.
Connections <i>Apprentice</i>	The student attempts to make a connection but it lacks contextual relevance.
Representation <i>Apprentice</i>	The student draws a diagram of the fenced area, but some of the unit lengths given for the sides are missing or incorrect. The diagrams units are not labeled.

Apprentice, Page 1

P/S	R/P	Com	Con	Rep	A/Level
A	A	P	A	A	A

Dear TMSFC,

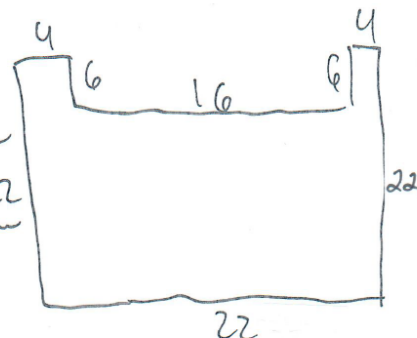
Thank you so much for the offer of installing our perimeter fencing for \$3,190, however I just cannot accept your offer.

When I went to find the perimeter I found all the units. I had a photo taken with a unit graph on top. I measured the different lengths (via counting) and got the following perimeter lengths: 22, 22, 4, 6, 16, 6, 4, 22 (All of these are units). I then added all of the units and got 105 units. Because I know that each unit of fencing costs \$0.89, I multiplied 105 by 0.89 and got \$93.45. \$93.45 would be the true cost of this perimeter fencing.

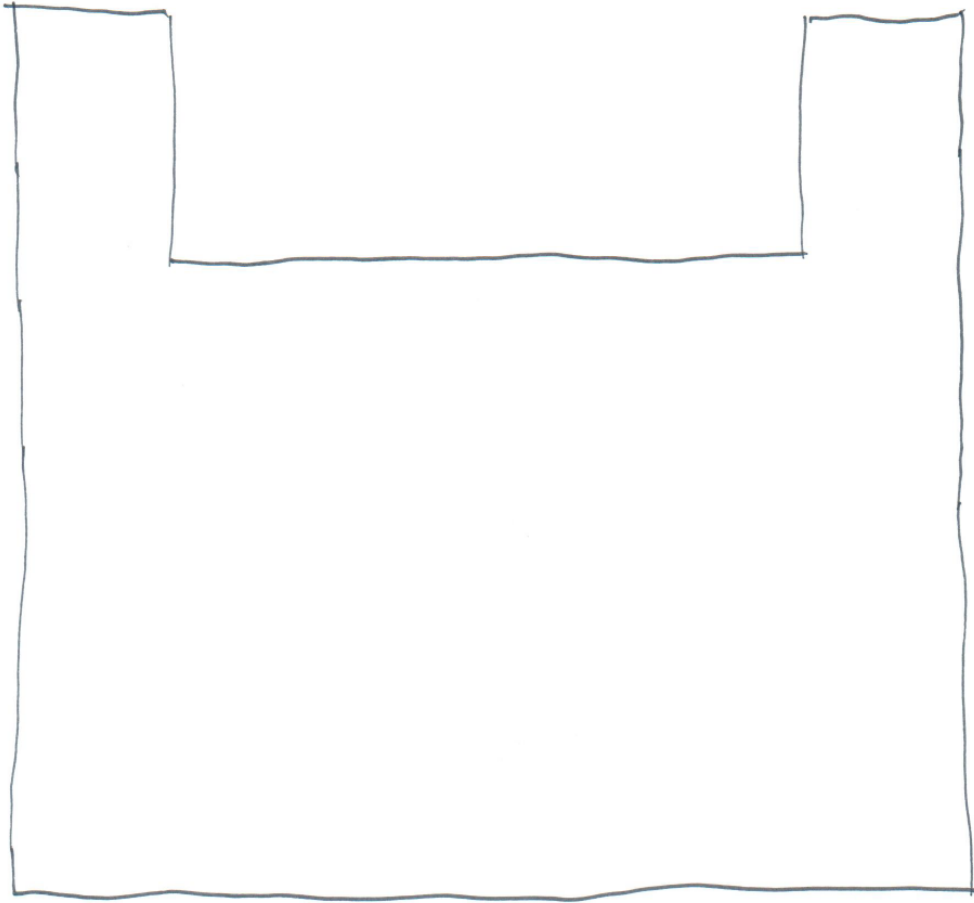
This is why I must decline your offer because \$93.45 is way less than \$3,190

Thank you again for your offer
- Mura

Connection / representation
I used counting, adding, and multiplication is this problem



Apprentice, Page 2



25 un, 22 un, 4 un, 6 un, 16 un, 6 un, 9 un,
 22 un = 105 units $\times 0.892 = 93.45$
 \$93.45 is a lot less than \$3140 that
 the TMSFC is offering

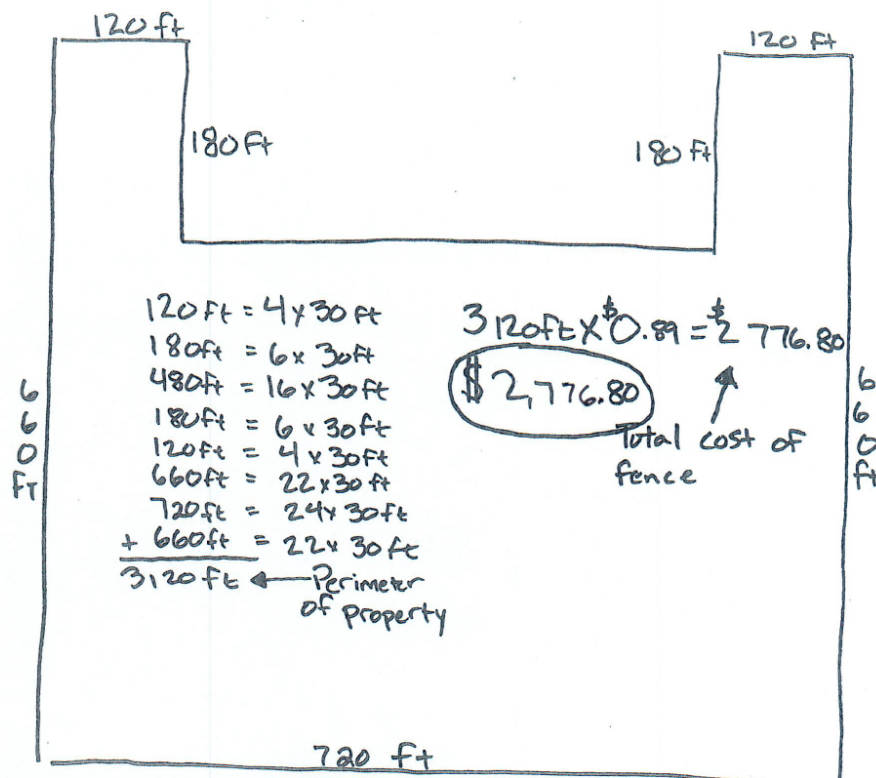
Practitioner Student 1 Scoring Rationale

Criteria and Performance Level	Rationales
Problem Solving <i>Practitioner</i>	The student correctly plots the points given to create a diagram of the fenced area. The student determines the unit lengths of each line segment and converts these to feet by multiplying each value by 30 (feet per unit length). The student correctly determines the cost at \$2776.80 by multiplying 3120 feet by \$.89 (cost/foot). The student correctly determines that the offer from the fence company should be rejected.
Reasoning & Proof <i>Practitioner</i>	The student's argument is sequenced, logical and easy to follow. The student demonstrates correct reasoning of the underlying concepts of the task. The student determines the length in feet of the fence and then calculates the total cost by multiplying by the unit rate of \$.89. Calculations are correct and support the solution given.
Communication <i>Practitioner</i>	The student uses an organized and sequenced response to communicate work. The student explains each step to the solution process. Work is clear and easy to follow. Appropriate math language is used.
Connections <i>Expert</i>	The student describes a second way the task can be solved (does not actually show the calculations). The student also makes a connection between the mathematics in the task and how the strategy for solving the task could be extended to other cases, "when your coding."
Representation <i>Practitioner</i>	The student plots the given points to create a diagram of the fenced area on a coordinate grid. The student correctly labels the length of each line segment in feet on the diagram.

Practitioner Student 1, Page 1

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	E	P	P

My first step was to put the coordinates on the grid. After that, I found out the side lengths of the shape/property. Once I had all the side lengths, I added them together to get the Perimeter. Once I got the perimeter/total footage I multiplied the number by 0.89 because each square foot costs 0.89 cents. The total cost was under the estimated Price which is why she should reject the offer. Also, to get the side lengths I multiplied 30ft by the number of squares on one side.



Marv should reject the offer because the real Price of the fence is under the estimated Price, which means she can save money.

Practitioner Student 1, Page 2

Dear Meow Safe Fencing Company

I am rejecting your offer because your charging me extra money for it. It would only cost \$2,776.80 because I found the perimeter of the property, and then multiplied that by 0.89 which is the amount of money it is for 1 square footage. This got me to the cost of \$2,776.80, which is a lot less than the cost your charging me.

Sincerely
Maru

Connection:

Another way we can solve it is multiplying each side length by 0.89, then adding them all together. But what I did was add the side lengths first then multiply it by 0.89. Also, this is the type of math you would also use when your coding like putting coordinates on a grid.

Practitioner Student 2 Scoring Rationale

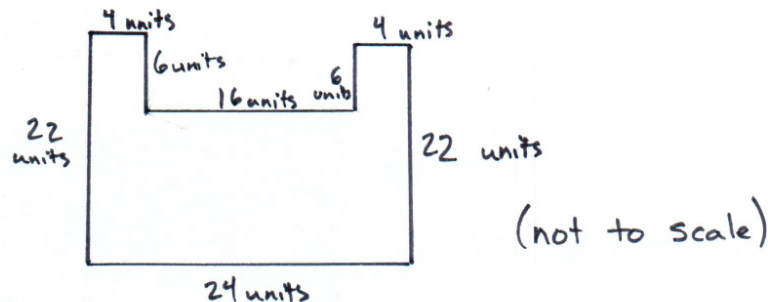
Criteria and Performance Level	Rationales
Problem Solving <i>Practitioner</i>	The student correctly plots the points given to create a diagram of the fenced area. The student determines the unit lengths of each line segment and converts these to feet by multiplying each value by 30 (feet per unit length). The student correctly determines the cost at \$2,776.80 by multiplying 3,120 feet by \$.89 (cost/foot). The student correctly determines that the offer from the fence company should be rejected.
Reasoning & Proof <i>Practitioner</i>	The student correctly plots points on a coordinate grid, finds the perimeter of the polygon created in unit lengths, and then converts units into feet in his diagram. Calculations for the total cost are present and correct. The student demonstrates an interesting strategy of finding half the total perimeter to enclose and then multiplying by 2 to find the other half of the perimeter.
Communication <i>Practitioner</i>	The student uses an organized, sequenced, and labeled response to communicate work. The student identifies the task to be solved, explains each step in the solution process, and explicitly states the solution. Units of measure are accurately recorded. The student uses appropriate math language.
Connections <i>Practitioner</i>	The student explores a mathematical phenomenon within their solution, "since this shape is symmetrical left to right, we can just find half the dimensions and multiply by 2." This secondary strategy recognizes an interesting pattern in finding perimeter of symmetrical shapes.
Representation <i>Expert</i>	The student creates two correct diagrams of the fenced area to help analyze the relationships between the different measurement for the perimeter of units versus feet.

Practitioner Student 2, Page 1

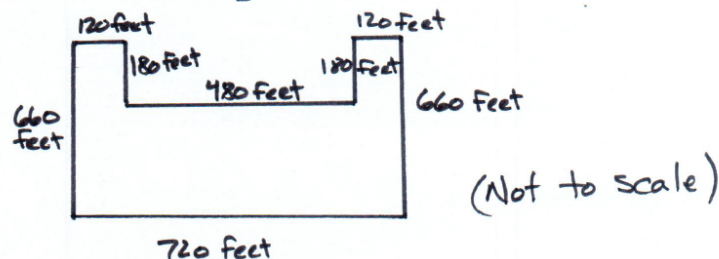
P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	E	P

Problem: Maru wants to buy a fence around her cat roaming area. A company charges \$.89 per foot and charges her \$3,190. Is that the correct price?

Roaming: We first have to draw the cat enclosure area with the coordinates given to us.
We get



But they say that 1 unit is 30 feet (30') so we have to multiply all the dimensions to get



Perimeter: Now we have to find the perimeter. We can do this by adding up all the sides or Since this shape is symmetrical left to right, we can just find half the dimensions and multiply by 2.

$$\begin{array}{r}
 660 \\
 + 120 \\
 180 \\
 240 (480 \div 2) \\
 360 (720 \div 2) \\
 \hline
 1560
 \end{array}$$

$$1560 \cdot 2 = 3120$$

So the perimeter is

3120 feet

Price: Now we can find the price and see if "meow safe fencing" charged the right amount. Each foot of fencing cost \$.89 and you have 3120 feet (perimeter) you can multiply them and find the price.

$$\begin{array}{r}
 \times 3120 \\
 .89 \\
 \hline
 \$2,776.80
 \end{array}$$

So Meow safe fencing overcharged Maru saying it would cost \$3190.

letter: The last part of the problem was to write a letter to the company accepting or rejecting their offer.

Dear Meow Safe Fencing,

I reject your offer and think you overpriced. I know this because the dimensions of my property are 720', 660', 120', 180', 480', 180', 120', 660' (see diagram in roaming area section.) If you add these up you get a perimeter of 3120 feet. Your company charges \$.89 a foot so you multiply 3120 by \$.89 to get a price of \$2,776.80. So you should change your offer from \$3,190 to \$2,776.80.

Expert Scoring Rationale

Criteria and Performance Level	Rationales
Problem Solving <i>Expert</i>	The student provides an effective strategy and a second more efficient strategy to solve the task. The alternative strategy at the end is considered which shows evidence of the student analyzing the situation and defining a more efficient strategy to solve the task, "but there is a different way..." The student then describes and diagrams this alternative strategy.
Reasoning & Proof <i>Expert</i>	The student provides a systematic mathematical justification throughout. Because the student also supports their idea mathematically for an alternative strategy on how to solve the task more efficiently, "compensate for the "dip" of 6 units at the top..." this student achieves Expert level work. Calculations are correct that support the solution.
Communication <i>Expert</i>	The student uses an organized, sequenced, and labeled response to communicate their work. The student provides insight into the efficiency of their original strategy and defines a more efficient method for reaching the same perimeter. Formal math language is used throughout to communicate their ideas.
Connections <i>Expert</i>	The student provides a deeper understanding of the mathematics in the task as they describe their alternative strategy to solve the task faster, "compensate for the "dip" of 6 units at the top..." Experts articulate connections between various strategies for solving the task.
Representation <i>Expert</i>	The student constructs a representation that clarifies their idea of using the perimeter of the rectangle "that this shape forms" plus the "the dip". The construction of the two representations helps to clarify how they can each be used to solve the overall question.

Expert Page 1

P/S	R/P	Com	Con	Rep	A/Level
E	E	E	E	E	E

Herding Cats 😊

Is the offer of \$3,190 fair for all fencing?

first I graphed the enclosure. I plotted the points given, then connected them so they formed a closed shape.

Then I measured the length (in units) of each side and added them together which ended up being 104 units (this is the perimeter.)

$$24 + 22 + 22 + 4 + 4 + 6 + 6 + 6 = \boxed{104}$$

After this, I converted the perimeter into feet by multiplying 104 by 30 (30 feet per unit) I got 3,120 feet.

Next, I multiplied 3,120 by .89 (\$.89 per foot of fence.) I got \$2,776.80 for the final price.

Finally, I compared the original estimated price of \$3,190 for the project to the actual price of \$2,776.80.

$$3,120 > 2,776.80$$

Expert Page 2

Based on these findings, I decided to reject the companies original offer of \$3,120 as it was not a fair price

Connection:

This year I remember graphing points and connecting them then finding the perimeter like this. I think that I would have done what I did here to find the perimeter, but there is a different way with this particular shape:

Take the side lengths of the rectangle (24x22) that this shape forms, but to compensate for the "dip" of 6 units of the top, just add 12 (6×2) to the original perimeter. You get the same answer, just faster.

