

## **WARNING!**

The resource you are about to see may LOOK like an ordinary  
**word problem or story problem.**

## **IT'S NOT**

**Exemplars are performance tasks... and they're far superior.**

### **Exemplars are POWERFUL**

They're real-world problems that KIDS get to decide how to solve. Our tasks are engineered to be solved in many different ways, so everyone can jump in and find a strategy that works for them. No hand-holding here. Kids get to think critically, be creative, and apply their math skills to authentic situations.

### **Exemplars are HIGHLY POTENT**

In the real world, math is everywhere! When solving Exemplars tasks, kids exercise their ENTIRE brains by practicing things like:



### **Math Solutions are THE KEY**

Are your students answer-getters? Not with Exemplars. Developing a mathematical solution is what builds lasting memories and a DEEP UNDERSTANDING of math concepts. And we'll teach you how to get there!

**Ready to build confidence, unlock your students' inner mathematicians, and celebrate all those 'aha' moments?**

**Let's Go!**

## A Guide to Exemplars Resources

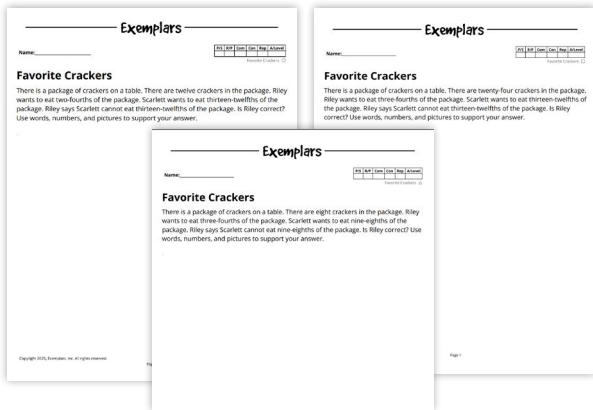
Exemplars problem-solving performance tasks are thoughtfully written and classroom-tested. Our rich tasks may be used for assessment, instruction, professional development, or to build a thinking classroom. Exemplars is the perfect supplement to your curriculum!

### Tasks Include:

#### Differentiated Versions

Standard Version

More Challenging Version



More Accessible Version

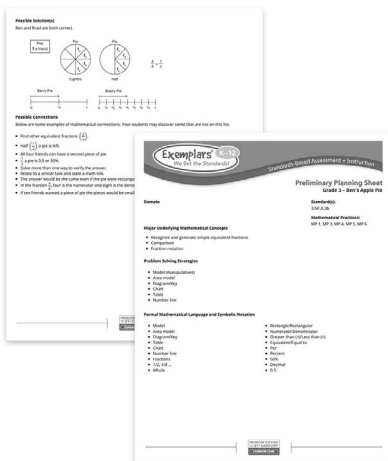
#### Engagement Images (to pique student curiosity)



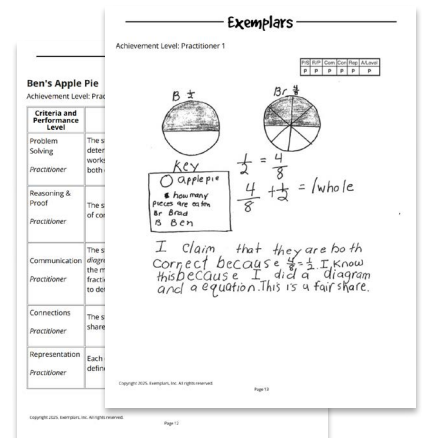
#### Standards-Based Math Rubric

Standards-Based Math Rubric	
Practitioner	Expert
<p><b>Problem Solving</b></p> <p>Identifies the problem and the information given. Plans a strategy to solve the problem. Uses a variety of strategies to solve the problem. Checks the solution.</p>	<p><b>Problem Solving</b></p> <p>Identifies the problem and the information given. Plans a strategy to solve the problem. Uses a variety of strategies to solve the problem. Checks the solution.</p>
<p><b>Reasoning &amp; Proof</b></p> <p>Explains the solution. Justifies the solution. Provides a clear explanation of the solution.</p>	<p><b>Reasoning &amp; Proof</b></p> <p>Explains the solution. Justifies the solution. Provides a clear explanation of the solution.</p>
<p><b>Connections</b></p> <p>Identifies the mathematical concepts involved. Makes connections between the concepts.</p>	<p><b>Connections</b></p> <p>Identifies the mathematical concepts involved. Makes connections between the concepts.</p>
<p><b>Representation</b></p> <p>Uses a variety of representations to solve the problem. Checks the solution.</p>	<p><b>Representation</b></p> <p>Uses a variety of representations to solve the problem. Checks the solution.</p>

#### Lesson Planning Sheets and Possible Solutions

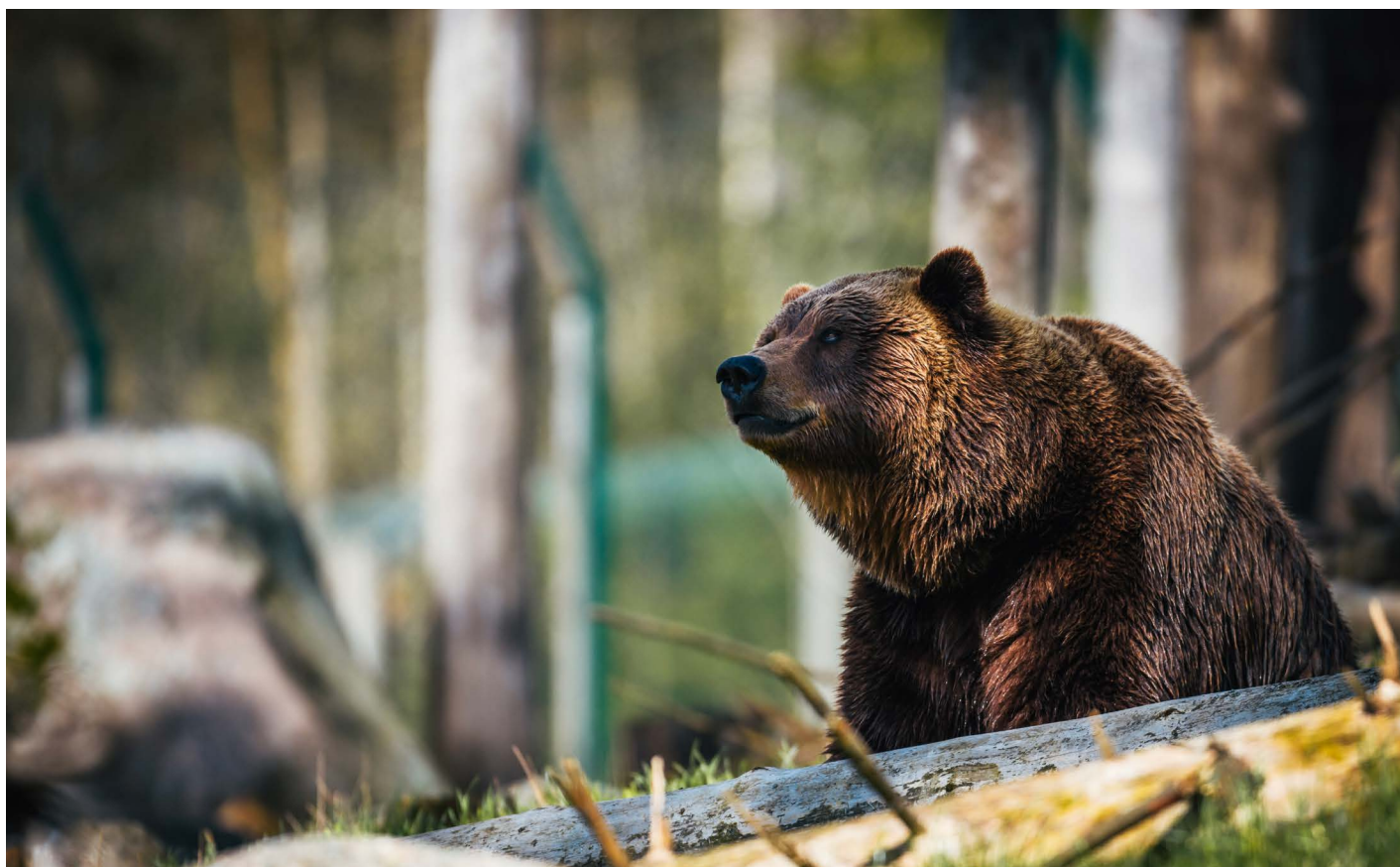


#### Scored Student Work Samples (examples of math solutions at 4 performance levels)



## Engagement Image to Launch Task

*Teachers use this resource to pique student curiosity.*



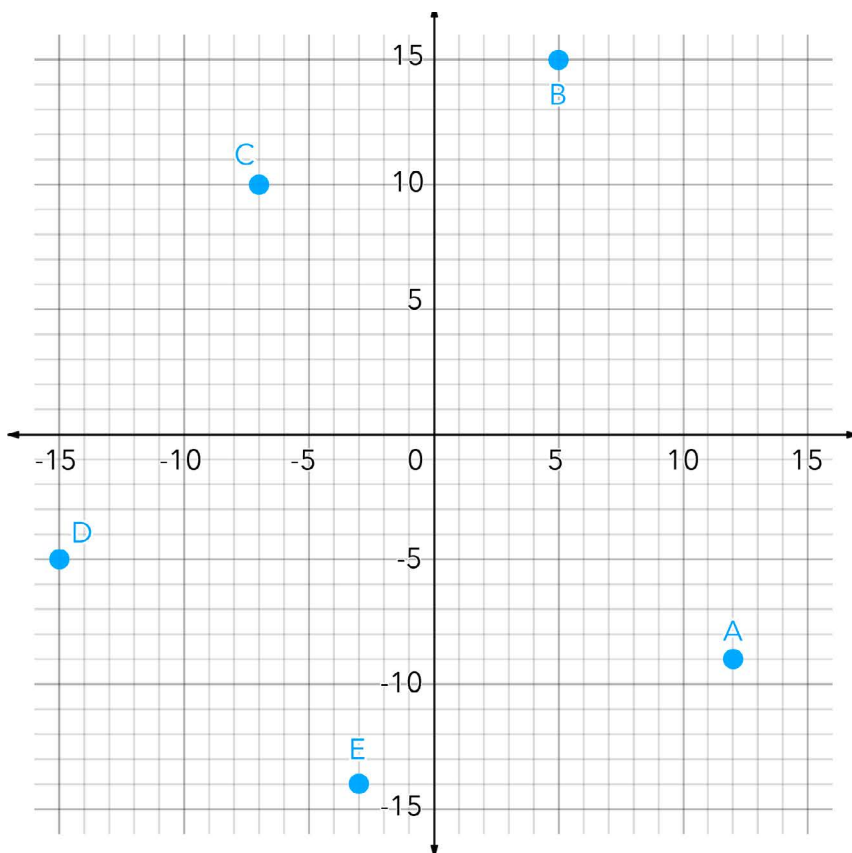
Instructional

## If a Bear Walks Into the Woods

Researchers in the Denali National Park in Alaska are using GPS collars to study the habits of wild grizzly bears in the park. Granny is a mother bear who researchers have been following for over 10 years. The locations on the grid below are the specific locations where Granny spent significant time during the previous day. Researchers wanted to use the data collected to approximate how far this grizzly bear may have traveled in a given day. Granny started and ended her day at location (12, -9) and moved from location to location in alphabetical order, as indicated on the grid.

Provide researchers with a clear explanation of your calculations for how far this grizzly may have traveled.

Note: One unit on the grid represents 500 ft.



# If a Bear Walks Into the Woods

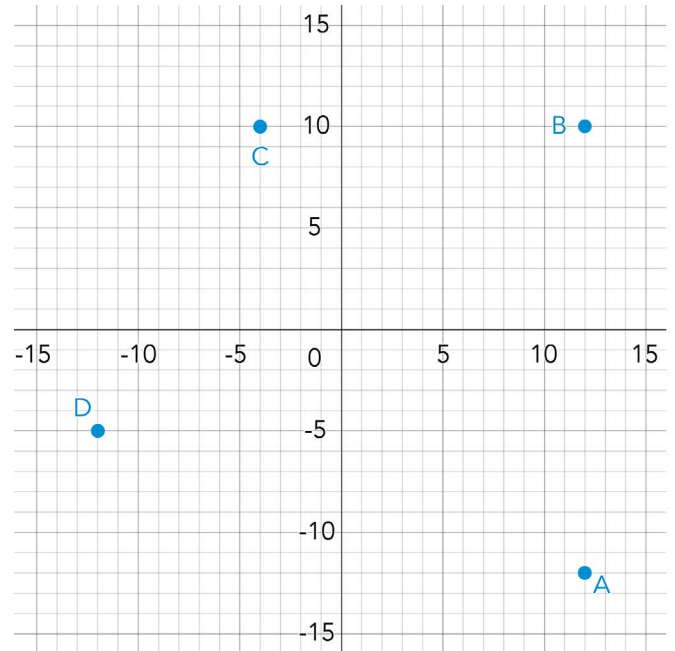
## Alternative Versions of the Task

### More Accessible Version

Researchers in the Denali National Park in Alaska are using GPS collars to study the habits of wild grizzly bears in the park. Granny is a mother bear who researchers have been following for over 10 years. The locations on the grid below are the specific locations where Granny spent significant time during the previous day. Researchers wanted to use the data collected to approximate how far this grizzly bear may have traveled in a given day. Granny started and ended her day at location (12, -12) and moved from location to location in alphabetical order, as indicated on the grid.

Provide researchers with a clear explanation of your calculations for how far this grizzly may have traveled.

Note: One unit on the grid represents 100 meters.



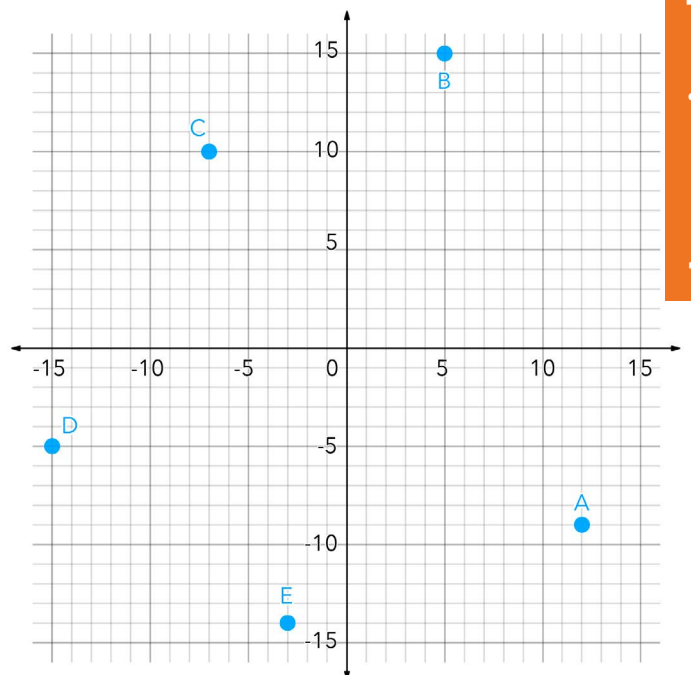
### More Challenging Version

Researchers in the Denali National Park in Alaska are using GPS collars to study the habits of wild grizzly bears in the park. Granny is a mother bear who researchers have been following for over 10 years. The locations on the grid below are the specific locations where Granny spent significant time during the previous day. Researchers wanted to use the data collected to approximate how far this grizzly bear may have traveled in a given day. Researchers are also trying to determine the total area of the territory Granny seems to utilize.

Granny started and ended her day at location (12, -9) and moved from location to location in alphabetical order, as indicated on the grid.

Provide researchers with a clear explanation of your calculations for how far this grizzly may have traveled and the area of her territory.

Note: One unit on the grid represents 500 ft.



Instructional

# Planning Sheet

## If a Bear Walks Into the Woods

### Common Core Task Alignments

**Mathematical Practices:** MP.2 MP.4 MP.6 MP.7

**Grade 8 Content Standards:** 8.G.B.8

### Common Core Standards and Evidence

#### 8.G.B.8

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

#### Exemplars Task-Specific Evidence

This task requires students to find the diagonal distance between points on a coordinate plane, using the Pythagorean Theorem, to determine how far a bear travels throughout its day.

### Underlying Mathematical Concepts

- Solving for unknowns in equations
- Scaling
- Pythagorean Theorem

### Possible Problem-Solving Strategies

- Solving for unknowns in equations
- Scaling
- Pythagorean Theorem

### Possible Mathematical Vocabulary/Symbolic Representation

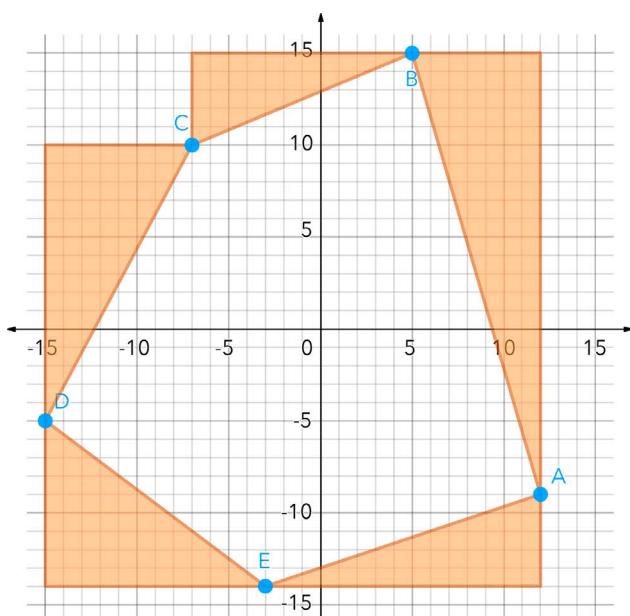
- |                       |                   |
|-----------------------|-------------------|
| • Average             | • Right angle     |
| • Constant rate       | • Right triangle  |
| • Coordinate plane    | • Scale           |
| • Coordinate point    | • Similar figures |
| • Exponent            | • Square root     |
| • Hypotenuse          | • Substitution    |
| • Legs                | • Sum             |
| • Perfect square      | • Unit rate       |
| • Pythagorean Theorem | • x-axis          |
| • Pythagorean triple  | • y-axis          |
| • Radicals            |                   |



## Possible Solutions

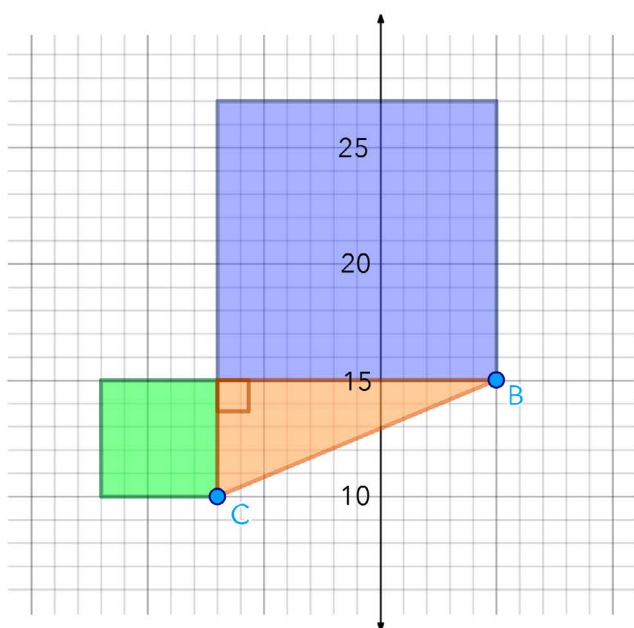
Granny traveled at least 42,900 feet.

The horizontal and vertical distance between points makes the legs of a right triangle between each set of



### Area Model of the Pythagorean Theorem

The relationship between the sides can be modeled on grid paper or using an open area.



	A to B	B to C	C to D	D to E	E to A
Longer Leg Square (square units)	$24 \bullet 24 = 576$	$24 \bullet 24 = 576$	$15 \bullet 15 = 225$	$12 \bullet 12 = 144$	$15 \bullet 15 = 225$
Shorter Leg Square (square units)	$7 \bullet 7 = 49$	$5 \bullet 5 = 25$	$8 \bullet 8 = 64$	$9 \bullet 9 = 81$	$5 \bullet 5 = 25$
Hypotenuse Square (square units)	$576 + 49 = 625$	$144 + 25 = 169$	$225 + 64 = 289$	$144 + 81 = 225$	$225 + 25 = 250$
Hypotenuse Length (units)	25	13	17	1	15.8

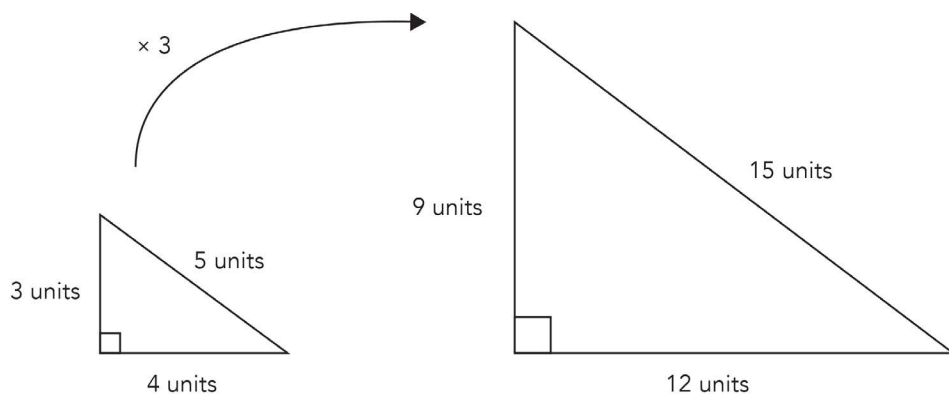
### Apply the Pythagorean Theorem

	A to B	B to C	C to D	D to E	E to A
Length of Longer Leg (units)	24	12	15	12	15
Length of Shorter Leg (units)	7	5	8	9	5
Pythagorean Theorem	$24^2 + 7^2 = c^2$ $576 + 49 = c^2$ $625 = c^2$	$12^2 + 5^2 = c^2$ $144 + 25 = c^2$ $169 = c^2$	$15^2 + 8^2 = c^2$ $225 + 64 = c^2$ $289 = c^2$	$12^2 + 9^2 = c^2$ $144 + 81 = c^2$ $225 = c^2$	$15^2 + 5^2 = c^2$ $225 + 25 = c^2$ $250 = c^2$
Hypotenuse (units)	25	13	17	15	15.8



### Similar Figures (for point D to E)

The length of the hypotenuse (c) has to be greater than the length of each leg (a and b).



(not drawn to scale)

### Guess and Check

The length of the hypotenuse (c) has to be greater than the length of each leg (a and b).

Point A to B

Longer Leg (a)	$a^2$	Shorter Leg (b)	$b^2$	Hypotenuse	$c^2$	$a^2 + b^2 = c^2$	Accuracy
24	576	7	49	25	625	$576 + 49 = 625$	Correct

Point B to C

Longer Leg (a)	$a^2$	Shorter Leg (b)	$b^2$	Hypotenuse	$c^2$	$a^2 + b^2 = c^2$	Accuracy
12	144	5	25	13	169	$144 + 25 = 169$	Correct

Point C to D

Longer Leg (a)	$a^2$	Shorter Leg (b)	$b^2$	Hypotenuse	$c^2$	$a^2 + b^2 = c^2$	Accuracy
15	225	8	64	16	256	$225 + 64 \neq 256$	Too Low
$\vdots$	$\vdots$	$\vdots$	$\vdots$			$225 + 64 = 289$	Correct

## Engagement Image to Launch Task

*Teachers use this resource to pique student curiosity.*



## Scary Reunion

A team of marine biologists are studying sharks in the Caribbean. The team has anchored their research vessel near the island of South Caicos. At 9 a.m., the team tagged two sharks with radio transmitters and released them. At 2 p.m., the team noted the location of the two sharks.

New locations:

Shark 1: Located 45 miles north and 60 miles west.

Shark 2: Located 36 miles south and 77 miles east.

If the team wanted to pull up the anchor at 2 p.m. and be in the same location as one of the sharks by 7 p.m., how fast would the research vessel need to travel to be in a location near one of the sharks? Assume each shark maintains its current speed and direction.

Provide a clear explanation of your calculations for the team.

# Planning Sheet

## Scary Reunion

### Common Core Task Alignments

**Mathematical Practices:** MP.1 MP.2 MP.4 MP.6

**Grade 6 Content Standards:** 8.G.B.8

### Common Core Standards and Evidence

#### 8.G.B.8

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

#### Exemplars Task-Specific Evidence

This task requires students to use the Pythagorean theorem to find how far two sharks will travel and how fast the vessel needs to go to catch up to either one.

### Underlying Mathematical Concepts

- Solving for unknowns in equations
- Pythagorean Theorem
- Finding and applying unit rates

### Possible Problem-Solving Strategies

- Pythagorean Theorem
- Area model

### Possible Mathematical Vocabulary/Symbolic Representation

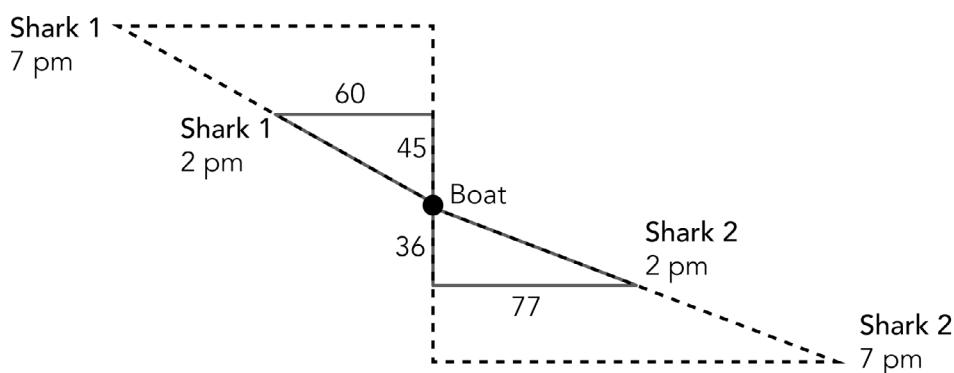
- |                  |                       |
|------------------|-----------------------|
| • Constant rate  | • Pythagorean Theorem |
| • Unit rate      | • Substitution        |
| • Perfect square | • Pythagorean triple  |
| • Right triangle | • Square root         |
| • Legs           | • Exponent            |
| • Hypotenuse     | • Sum                 |

## Possible Solutions

The research vessel needs to travel 30 miles per hour to be in the same location as Shark 1 or 34 miles per hour to be in the same location as Shark 2 at 7 p.m.

Students may use a variety of strategies and solution paths to determine how far each shark is from the boat at 2 p.m. and how fast the vessel needs to travel to reach the sharks at 7 p.m.

The solution below is based on each shark's given location and the time they travel in 5 hours. Their location at 7 p.m. (10 hours) could also be used instead and is just double the distances for 5 hours.



Shark 1 traveled 75 miles in 5 hours.

Shark 2 traveled 85 miles in 5 hours.

### Distance from the Boat to each Shark's Location

#### Apply the Pythagorean Theorem

Shark 1

$$a^2 + b^2 = c^2$$

$$60^2 + 45^2 = c^2$$

$$3,600 + 2,025 = c^2$$

$$5,625 = c^2$$

$$c = \sqrt{5,625} = 75 \text{ miles}$$

Shark 2

$$a^2 + b^2 = c^2$$

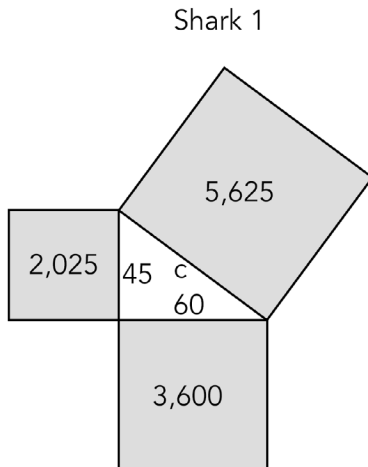
$$36^2 + 77^2 = c^2$$

$$1,296 + 5,929 = c^2$$

$$7,225 = c^2$$

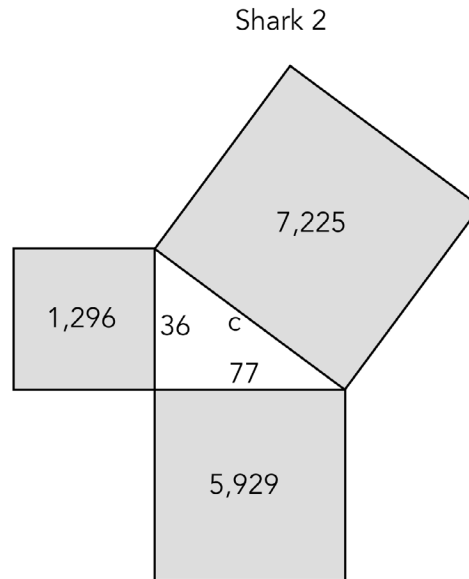
$$c = \sqrt{7,225} = 85 \text{ miles}$$

## Apply the Pythagorean Theorem



$$3,600 + 2,025 = 5,625$$

$$c = \sqrt{5,625} = 75 \text{ miles}$$



$$1,296 + 5,929 = 7,225$$

$$c = \sqrt{7,225} = 85 \text{ miles}$$

## Speed of the Research Vessel

The sharks will travel for another 5 hours at their current rate. This means the vessel needs to travel twice as fast as the sharks to catch up to them.

OR

Shark 1

75 miles  $\div$  5 hours = 15 miles per hour

15 mph  $\times$  10 hours = 150 miles

Vessel speed: 150 miles  $\div$  5 hours = 30 miles per hour

Shark 2

85 miles  $\div$  5 hours = 17 miles per hour

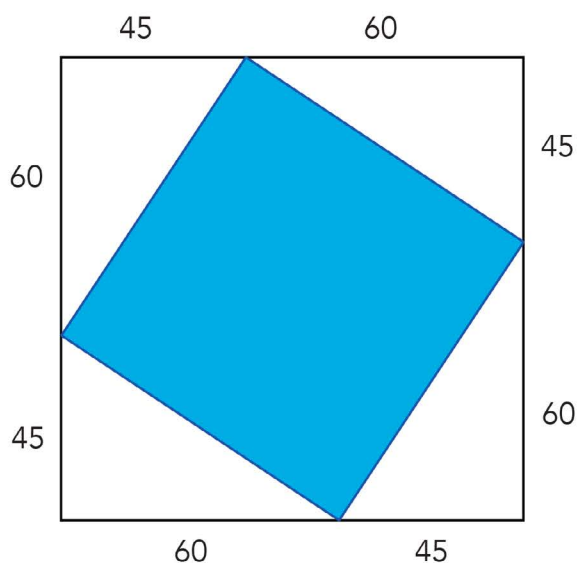
17 mph  $\times$  10 hours = 170 miles

Vessel speed: 170 miles  $\div$  5 hours = 34 miles per hour



## Surround and Subtract

Shark 1



Area of Large Square:

$$105 \cdot 105 = 11,025 \text{ square miles}$$

Area of Each Triangle:

$$(45 \cdot 60) \div 2 = 1,350 \text{ square miles}$$

Area of all 4 Triangles:

$$1,350 \cdot 4 = 5,400 \text{ square miles}$$

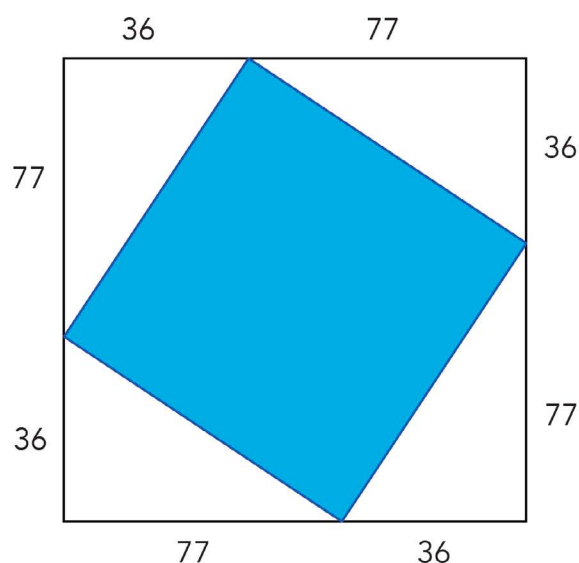
Area of Hypotenuse Square (shaded area):

$$11,025 - 5,400 = 5,625 \text{ square miles}$$

Side Length of Hypotenuse Square ( $c$ )

$$= \sqrt{5,625} = 75 \text{ miles}$$

Shark 2



Area of Large Square:

$$113 \cdot 113 = 12,769 \text{ square miles}$$

Area of Each Triangle:

$$(36 \cdot 77) \div 2 = 1,386 \text{ square miles}$$

Area of all 4 Triangles:

$$1,386 \cdot 4 = 5,544 \text{ square miles}$$

Area of Hypotenuse Square (shaded area)

$$12,769 - 5,544 = 7,225 \text{ square miles}$$

Side Length of Hypotenuse Square ( $c$ )

$$= \sqrt{7,225} = 85 \text{ miles}$$

## Possible Connections

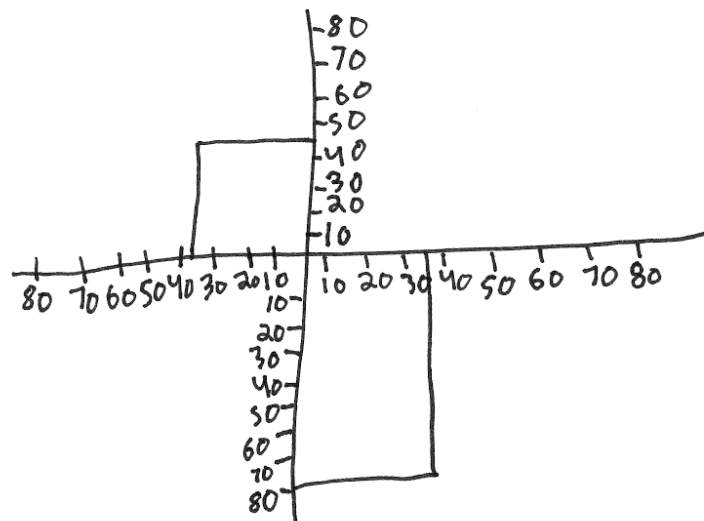
- The Meow Safe Fencing Company's estimate is \$423.20 more than the expected cost.
- Determine the area of the roaming space (432 sq units or 388,800 sq ft).
- The roaming area has a perimeter of 1,040 yards.
- The Meow Safe Fencing Company charges \$2.67 per yard of electric fencing.
- Maru could maximize the area and spend less money on fencing if she made the roaming area a rectangle instead of an irregular shape.
- Relate to a similar task and state a math link.
- Cost for fencing can be stated algebraically:  $C = 0.89f$ .
- $C$  = total cost of fencing for a property
- $f$  = total feet needed for fencing

## Novice Scoring Rationale

Criteria and Performance Level	Rationales
<b>Problem Solving</b> <i>Apprentice</i>	The student's strategy of constructing a graph to show the location of each shark would work but the student does not correctly plot the points nor calculate the actual distance traveled. The students answers of "a team of marine biologists will need to travel 45 miles per hour to be in the same location of shark 1" and "the marine biologists will need to travel 51 miles per hour to see shark 2" are incorrect.
<b>Reasoning &amp; Proof</b> <i>Novice</i>	The student does not demonstrate understanding of the underlying concepts of finding the distance traveled using a coordinate plane or the Pythagorean Theorem. There is no justification for reasoning present and arguments are made with no mathematical basis.
<b>Communication</b> <i>Apprentice</i>	The appropriate use of formal math language is minimal with miles per hour and graph. The student makes no attempt to provide a written account of their approach for finding 45 miles per hour or 51 miles per hour.
<b>Connections</b> <i>Novice</i>	The student does not make a mathematical connection about their solution.
<b>Representation</b> <i>Apprentice</i>	The student attempts to make a coordinate graph but does not accurately plot the location of shark 1, does not include labels, does not use it to find the distance or speed traveled.

Novice

P/S	R/P	Com	Con	Rep	A/Level
A	N	A	N	A	N



The team of Marine Biologists will need to travel 45 miles per hour to be in the same location of shark 1. The Marine Biologists will need to travel 51 miles an hour to see shark 2. That is if they wanted to be in the same location at 7 P.M. because of my work that I showed above with the graph. Shark 1- 45 mph  
Shark 2- 51 mph

## Apprentice Student 1 Scoring Rationale

Criteria and Performance Level	Rationales
<b>Problem Solving</b> <i>Apprentice</i>	The student's strategy of using the Pythagorean Theorem works to solve the first part of the task. The student's answer for the first part of the task is correct, "Shark #2 @ 2 pm = 17 mph" and "Shark #1 @ 2 pm = 15 mph." The student provides no evidence of their strategy that therefore converting the Shark #1 speed of 15 mph to the boat needing to go 30 mph.
<b>Reasoning &amp; Proof</b> <i>Apprentice</i>	The student demonstrates correct reasoning for some of the underlying concepts of the task, such as using the Pythagorean Theorem to calculate the distance traveled by each shark at 2 pm, " $36^2 + 77^2 = 7225$ , $\sqrt{7225} = 85$ " and " $45^2 + 60^2 = 5625$ , $\sqrt{5625} = 75$ ." The student does not show correct reasoning for calculating the speed of shark or vessel. The student does not provide mathematical justification for " $85 \div 5 = 17$ mph", " $75 \div 5 = 15$ mph", and "they'll have to go 30 miles per hour."
<b>Communication</b> <i>Apprentice</i>	The student attempts to make an organized and sequenced response, but it is incomplete. The does not communicate their approach including the use of the Pythagorean Theorem, the formula for speed, or why the boat will have to go 30 mph. The appropriate use of formal math language is minimal including fast, mph, slower, miles per hour.
<b>Connections</b> <i>Novice</i>	The student's connection is contextually irrelevant, "Shark One is slower so it will be easier to catch" as it does not explore the concept of the formula for speed.
<b>Representation</b> <i>Apprentice</i>	The student attempts to construct a coordinate grid to investigate the location of the shark after 5 hours. The student does not provide any labels for the data provided on the grid or for the units on the x and y coordinates.

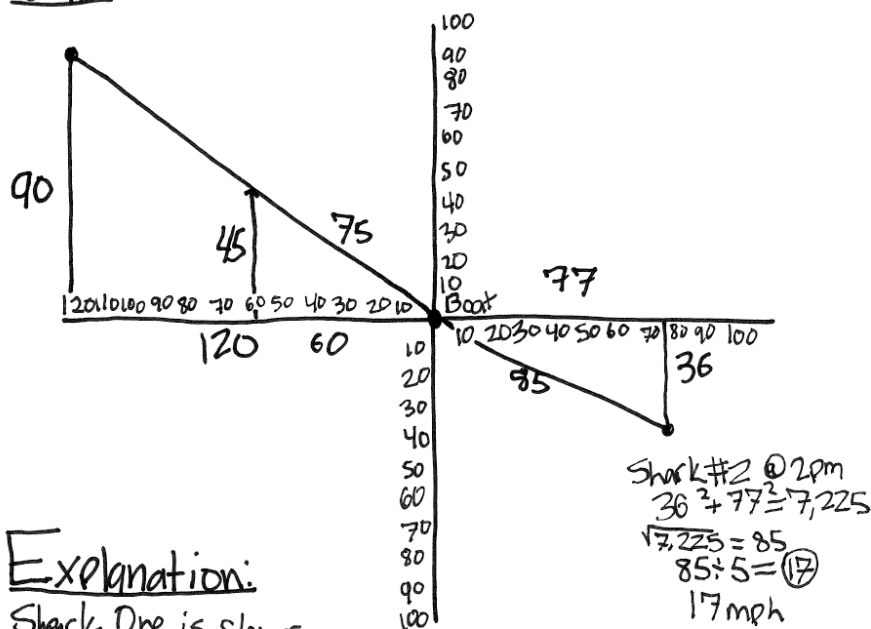
Apprentice Student 1

P/S	R/P	Com	Con	Rep	A/Level
A	A	A	N	A	A

## Scary Reunion

Question Statement: How fast does the team need to in order to catch up to the sharks?

What I knew: Two sharks were tagged at 9am  
Work:



Explanation:

Shark One is slower so it will be easier to catch.  
 If they're going to catch the shark at 7 they'll have to go 30 miles per hour.

Shark #1 @ 2pm  
 $45^2 + 60^2 = 5,625$   
 $\sqrt{5,625} = 75$   
 $75 \div 5 = 15$   
 15 mph



## Apprentice Student 2 Scoring Rationale

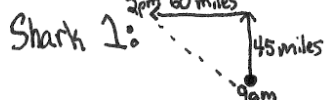
Criteria and Performance Level	Rationales
<b>Problem Solving</b> <i>Apprentice</i>	The student makes mistakes in determining how far the sharks traveled. The student correctly uses the Pythagorean Theorem for incorrect values of the distances traveled by each shark. The student correctly uses the formula for speed but arrives at an incorrect answer due to their previous mistake. The student incorrectly states "The boat would have to go 17.1 or 17 mph to be near shark 1 at 7 pm" and "The boat would have to go 19.3 or 19 mph to be near shark 2 at 7 pm."
<b>Reasoning &amp; Proof</b> <i>Practitioner</i>	The student demonstrates correct reasoning of the Pythagorean Theorem, " $452 + 1652 = c^2$ " and " $362 + 1902 = c^2$ ". The student correctly uses the formula for speed, "speed = distance $\div$ time" to provide mathematical justification for the speed of the sharks, " $171 \div 10 = 17.1$ mph", and " $193 \div 10 = 19.3$ mph."
<b>Communication</b> <i>Apprentice</i>	The student attempts to construct an organized, sequenced and labeled response but is inconsistent on defining each step of their approach. The student does not explain why they used " $60 + 45 = 105$ " and " $36 + 77 = 113$ " to determine how far each shark traveled in the first five hours.
<b>Connections</b> <i>Apprentice</i>	The student attempts to make a mathematically relevant connection, "I realized that I need to find the hypotenuse first." This connection is relevant but does not explore or clarify how this would change their approach or final answer.
<b>Representation</b> <i>Apprentice</i>	The student constructs multiple diagrams to help them solve the problem and to portray their thinking. The diagrams appropriately and accurately represent the student's approach that illustrates where mathematical misconceptions occurred.

## Apprentice Student 2

P/S	R/P	Com	Con	Rep	A/Level
A	P	A	A	P	A

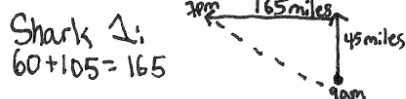
### Scary Revision

~ The question is asking me how fast a research vessel would have to travel if they wanted to be in the same place as a Shark by 7 pm.



$60 + 45 = 105 \rightarrow$  Shark traveled 105 miles in 5 hours.

★ Sharks continue at the same



$60 + 105 = 165$

$$45^2 + 165^2 = c^2$$

$$2025 + 27225 = c^2$$

$$\sqrt{29250} = \sqrt{c^2}$$

$$c \approx 171$$

$$a = 45$$

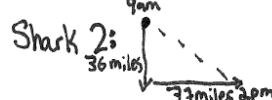
$$b = 165$$

$$c = ?$$

$$? \approx 171$$

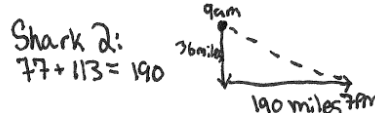
Shark 1: 171 miles NW

distance/time =  $171 \div 10 = 17.1$  in 10 hours  
 $17.1 \div 10 = 17.1$  mph



$36 + 77 = 113 \rightarrow$  Shark traveled 113 miles in 5 hours.

★ Speed and Same Direction ★



$77 + 113 = 190$

$$36^2 + 190^2 = c^2$$

$$1296 + 36100 = c^2$$

$$\sqrt{37396} = \sqrt{c^2}$$

$$c \approx 193$$

$$a = 36$$

$$b = 190$$

$$c = ?$$

$$? \approx 193$$

Shark 2: 193 miles SE

distance/time =  $193 \div 10 = 19.3$  in 10 hours  
 $19.3 \div 10 = 19.3$  mph

Pythagorean theorem:  $a^2 + b^2 = c^2$   
Speed = distance/time

The boat would have to go 17.1 or 17 mph to be near shark 1 at 7 pm.

The boat would have to go 19.3 or 19 mph to be near shark 2 at 7 pm.

I realized that I need to find the hypotenuse first, this is rough draft/first draft.

## Practitioner Student 1 Scoring Rationale

Criteria and Performance Level	Rationales
<b>Problem Solving</b> <i>Practitioner</i>	The student's strategy of plotting the position of each shark on a coordinate grid and using the Pythagorean Theorem to calculate the distance traveled in the first 5 hours works to solve the task. The student's answer of the boat needing to travel 30 mph to catch shark 1 and 34 mph to catch shark 2 is correct.
<b>Reasoning &amp; Proof</b> <i>Practitioner</i>	The student shows understanding that the Pythagorean Theorem can be used to calculate the distance each shark traveled, " $45^2 + 60^2 = 5,625 = c^2$ , $c = 75$ miles." The student correctly calculates the speed of each shark, " $75 \div 5 = 15$ mph" and " $85 \div 5 \text{ hrs} = 17$ mph" and then determines the speed of the boat "which is double the distance at 2 pm."
<b>Communication</b> <i>Practitioner</i>	The student correctly identifies the problem to be solved in their opening statement, describes their approach in an organized and coherent response, and states a correct conclusion. Appropriate formal math language such as Pythagorean Theorem, mph, distance, graph, double, constant rate is used to share and clarify ideas.
<b>Connections</b> <i>Expert</i>	The student makes a mathematically relevant connection by exploring the concept of a "constant rate of 15 mph". The student uses this connection to determine to double the speed of the boat to catch the shark in 5 hours.
<b>Representation</b> <i>Practitioner</i>	The student's coordinate grid is appropriate and accurate for determining the location of the shark after 5 hours and after 10 hours. The student uses the coordinate grid to analyze the relationship between the distance traveled in the first 5 hours and in the second five hours. All necessary labels are provided and the entered data is correct.

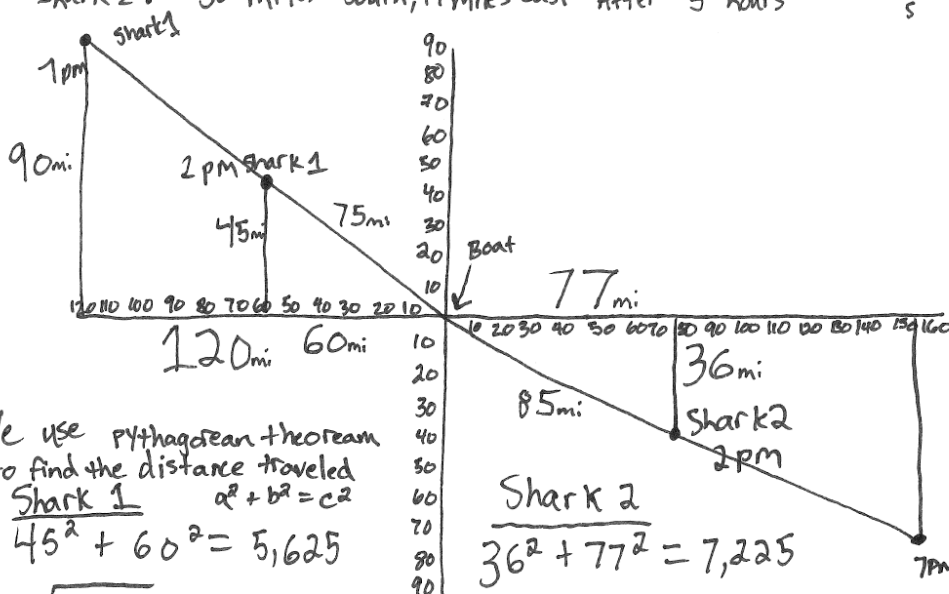
Practitioner Student 1

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	E	P

## Scary Reunion

How fast would the research vessel need to travel to be in a location near one of the sharks by 7 PM?

Shark 1: 45 miles north, 60 miles west After 5 hours  
Shark 2: 36 miles South, 77 miles east After 5 hours



We use Pythagorean theorem to find the distance traveled

Shark 1  $a^2 + b^2 = c^2$   
 $45^2 + 60^2 = 5,625$

$\sqrt{5,625} = 75 \text{ miles}$

$75 \div 5 = 15$

Speed of Shark 1 = 15 mph

(We divide by 5  
bc this is the location  
of the shark after 5 hours)

(speed is distance (miles)  
divided by time (hours))

Shark 2

$36^2 + 77^2 = 7,225$

$\sqrt{7,225} = 85 \text{ miles}$

$85 \div 5 = 17$

Speed of Shark 2 =

17 mph

## Practitioner Student 2 Scoring Rationale

Criteria and Performance Level	Rationales
<b>Problem Solving</b> <i>Practitioner</i>	The student's strategy of drawing diagrams to show the shark's journey, using the Pythagorean Theorem to calculate the sharks' distance traveled in 5 hours, and the formula for speed effectively solves the task. The student's answer of needing to travel 30 mph to be near shark 1 and to go 34 mph to be near shark 2 is correct.
<b>Reasoning &amp; Proof</b> <i>Practitioner</i>	The student correctly applies the Pythagorean Theorem to calculate the distance traveled by each shark, " $60^2 + 45^2 = c^2$ , $c = 75$ miles" and " $362 + 77^2 = c^2$ , $c = 85$ miles". The student recognized that they needed to "double(d) their distance" to determine the total distance traveled by the boat. The student also correctly uses the formula for finding "speed = distance $\div$ time" to correctly find how fast the boat needs to travel in 5 hours.
<b>Communication</b> <i>Practitioner</i>	The student correctly identifies the problem, describes the steps to the solution, and states a correct conclusion in the last paragraph. Appropriate formal math language such as Pythagorean Theorem, speed, distance, time, hypotenuse, right triangles, $90^\circ$ turns, formula, doubled, mph are used to share and clarify ideas.
<b>Connections</b> <i>Practitioner</i>	The student notes the regularity that "they swam that distance in 5 hours (9 am - 2 pm), but they actually had 10 hours (9 am - 7 pm) to swim so I doubled their distance." The student makes an important observation that sharks "wouldn't make $90^\circ$ turns, so I found the hypotenuse, a more realistic path." The student makes a connection with what they understand in the real world about swimming behavior.
<b>Representation</b> <i>Practitioner</i>	The student uses a diagram to show the distances and directions the sharks traveled. The diagrams are labeled correctly and show that the distance traveled by the shark is likely the hypotenuse between 2 points.

Practitioner Student 2

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

Pythagorean Theorem:  $a^2 + b^2 = c^2$

Scary Reunion

Speed = distance ÷ time

~ The question is asking me how fast a research vessel would have to go to be near a shark at 7 pm. I will use Pythagorean theorem to determine the distance traveled.

**Shark 1:** 2pm 60 miles

$a = 60$   
 $b = 45$   
 $c = ?$   
 $? = 75$

$$60^2 + 45^2 = c^2$$

$$3600 + 2025 = c^2$$

$$\sqrt{5625} = \sqrt{c^2}$$

$$c = 75 \text{ miles}$$

Shark 1 went 75 miles in 5 hours.  
That means it went 150 miles in 10 hours.

Speed =  $150 \div 5$   
Speed = 30 mph

**Shark 2:** 9am

$a = 36$   
 $b = 77$   
 $c = ?$   
 $? = 85$

$$36^2 + 77^2 = c^2$$

$$1296 + 5929 = c^2$$

$$\sqrt{7225} = \sqrt{c^2}$$

$$c = 85 \text{ miles}$$

Shark 2 went 85 miles in 5 hours.  
That means it went 170 miles in 10 hours.

Speed =  $170 \div 5$   
Speed = 34 mph

The boat would have to go 30 mph to be near shark 1 at 7 pm. The boat would have to go 34 mph to be near shark 2 at 7 pm.

To find my answers I had to do a number of things. First, I drew diagrams of each shark's journey. I found that they made right triangles. Since the sharks wouldn't make 90° turns, I found the hypotenuse, a more realistic path. They swam that distance in 5 hours (9am-2pm), but they actually had 10 hours (9am-7pm) to swim, so I doubled their distance. The boat only had 5 hours to reach a shark (2pm-7pm) so I divided each distance by 5. I did that because that is the formula for speed, as shown at the top of the page. That got me my two answers. 30 mph to reach Shark 1, and 34 mph to reach Shark 2.



## Practitioner Student 3 Scoring Rationale

Criteria and Performance Level	Rationales
<b>Problem Solving</b> <i>Practitioner</i>	The student's strategy of using a coordinate grid helps them recognize they need "to find the diagonal distance for each shark." The student uses the Pythagorean Theorem to calculate the diagonal distance of each shark at 2 pm, the formula for speed = distance/time to calculate the shark's speed, and then "you double the sharks speed because the vessel goes the same distance at half the time." The students answer that "for shark #1 the vessel must go 30 mph whist the vessel would have to go 34 mph to reach shark #2" is correct.
<b>Reasoning &amp; Proof</b> <i>Practitioner</i>	The student's arguments are constructed with adequate mathematical basis. The student correctly applies the Pythagorean Theorem to calculate the distance traveled by each shark, " $60^2 + 45^2 = c^2$ , $c = 75$ miles" and " $36^2 + 77^2 = c^2$ , $c = 85$ miles." The student correctly justifies the speed of each shark, " $75 \text{ miles} \div 5 \text{ hours} = 15 \text{ mph}$ , $85 \text{ miles} \div 5 \text{ hours} = 17 \text{ mph}$ ." The student also shows correct reasoning for how fast the boat would need to travel by doubling the shark's speed over 5 hours to catch up with the shark in 10 hours.
<b>Communication</b> <i>Practitioner</i>	A sense of purpose is communicated by the student in the original Question section, "If they wanted to pull up the anchor a 2 pm and end up near one of the sharks, how fast would the vessel have to average?" The student's approach is provided within the Explanation, "First I had to find the Diagonal distance for each shark" and "I then used the formula for speed = D/T to find each SHARKS speed." Appropriate formal math language such as average, location, speed, distance, time, "diagonal distance", "pathgarum therum", right triangle, formula, double, mph are used to share and clarify ideas.
<b>Connections</b> <i>Practitioner</i>	The student solves the tasks and notes the pattern "the sharks distance forms a right triangle." The student explores the relationship "the vessel needs to travel double the speed the sharks go."
<b>Representation</b> <i>Practitioner</i>	The student's use of a coordinate grid and compass rose to illustrate the position of each shark after 5 hours is appropriate and accurate. All necessary labels are provided and the information is correct.

Practitioner Student 2

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

## Scary Reunion

### Question

If they wanted to pull up the anchor at 2 pm and end up near one of the sharks, how fast would the vessel have to average?

### Information needed

#### Shark locations

- Shark 1
  - 45 miles N & 60 miles W
- Shark 2
  - 36 miles S & 77 miles E

- tagged at 9 am
- Pulled anchor @ 2 pm
- Meets one of the sharks @ 7 pm
- All 5 hour difference
- Speed =  $\frac{\text{Distance}}{\text{Time}}$
- $A^2 + B^2 = C^2$  (Pythagorean theorem)

C = The Diagonal Distance the shark travels

Shark #1  
 $60^2 + 45^2 = C^2$   
 $3600 + 2025 = C^2$   
 $5625 = C^2$   
 $\sqrt{5625} = \sqrt{C^2}$   
 $C = 75 \text{ miles}$

Shark #2  
 $36^2 + 77^2 = C^2$   
 $1296 + 5929 = C^2$   
 $7225 = C^2$   
 $\sqrt{7225} = \sqrt{C^2}$   
 $C = 85 \text{ miles}$

I noticed that you could use the Pythagorean theorem because the sharks' distance forms a right triangle.

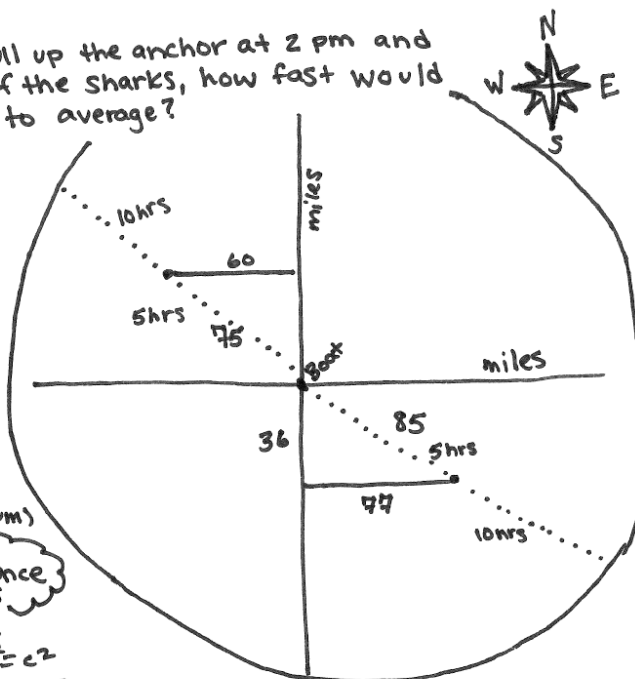
#1  $75 \text{ miles} \div 5 \text{ hours} = 15 \text{ mph}$  speed of shark  
 #2  $85 \text{ miles} \div 5 \text{ hours} = 17 \text{ mph}$  speed of shark

$15 + 15 = 30$   
 $17 + 17 = 34$   
 I divided by 5 because 9 am - 2 pm is 5 hrs and that's how long the sharks traveled for

You double the sharks speed because the vessel goes the same distance in half the time

### Explanation:

First I had to find the Diagonal distance for each Shark. Then I used the known fact that the times are 5 hrs apart. I then used the formula for  $\text{Speed} = \frac{D}{T}$  to find each Shark's speed. Since the boat starts late, and the sharks go double the distance, the vessel needs to travel double the speed the sharks go. This ultimately gives 30 mph (Shark 1) and 34 mph (Shark 2). This is the speed the vessel needs to travel.



Answer, For shark #1 the vessel must go 30 mph whilst the vessel would have to go 34 mph to reach Shark #2

## Expert Scoring Rationale

Criteria and Performance Level	Rationales
<b>Problem Solving</b> <i>Expert</i>	The student's strategy of drawing a diagram using the provided directional information works to find the distance traveled by the sharks in 5 hours and 10 hours. The student uses their diagram to show that there would be two congruent triangles creating a doubling of the distance traveled. The student's alternate strategy uses the Pythagorean Theorem to calculate the distance the shark traveled in the first 5 hours and then doubling it to find its distance at 7 pm. The student's answer of the boat needing to travel 30 mph to catch shark 1 and 34 mph to catch shark 2 is correct. The student recognizes that the shark is likely to travel twice the distance in twice the time to determine how far the shark will have traveled by 7 pm.
<b>Reasoning &amp; Proof</b> <i>Expert</i>	The student demonstrates correct reasoning by using Pythagorean theorem to find distance traveled on a coordinate grid, $a^2 + b^2 = c^2$ . The student also uses the speed formula to find the correct speed the boat will need to travel to catch both sharks in 5 hours, "Shark 1: $150/5 = 30$ , 30 miles per hour to catch the shark" and "Shark 2: $170/5 = 34$ , 34 miles per hours to catch the shark. The student utilizes their diagram as evidence to support their conclusion of the distance traveled by the shark at 5 hours and 10 hours.
<b>Communication</b> <i>Expert</i>	The student uses a methodical, organized and sequenced response to communicate their approach. The student correctly identifies the problem, describes the steps to the solution, and states a correct conclusion. Insight is communicated about an efficient strategy when the student states "The sharks will double their distance by 7 pm." The student utilizes their insight to construct the congruent triangles and to move from " $x = 75$ miles" to " $2x = 150$ miles" to find the total distance needed to travel. Appropriate math language, such as diagram, location, difference, double, distance, Pythagorean Theorem, hypotenuse, miles, time, miles per hour, speed is used rigorously to share and clarify ideas.
<b>Connections</b> <i>Expert</i>	The student uses several alternative strategies for finding the distance traveled and the speed required to make a mathematically relevant math connection. The student explains the phenomena that "the boat has to travel at double the speed of the sharks, because it travels double the distance in the same amount of time."
<b>Representation</b> <i>Expert</i>	The student analyzes the relationships between the distance the sharks travel in 5 hours and 10 hours by using congruent triangles to show the total distance traveled. The student uses the diagram to clarify that doubling the time doubles the distance traveled.

## Expert Page 1

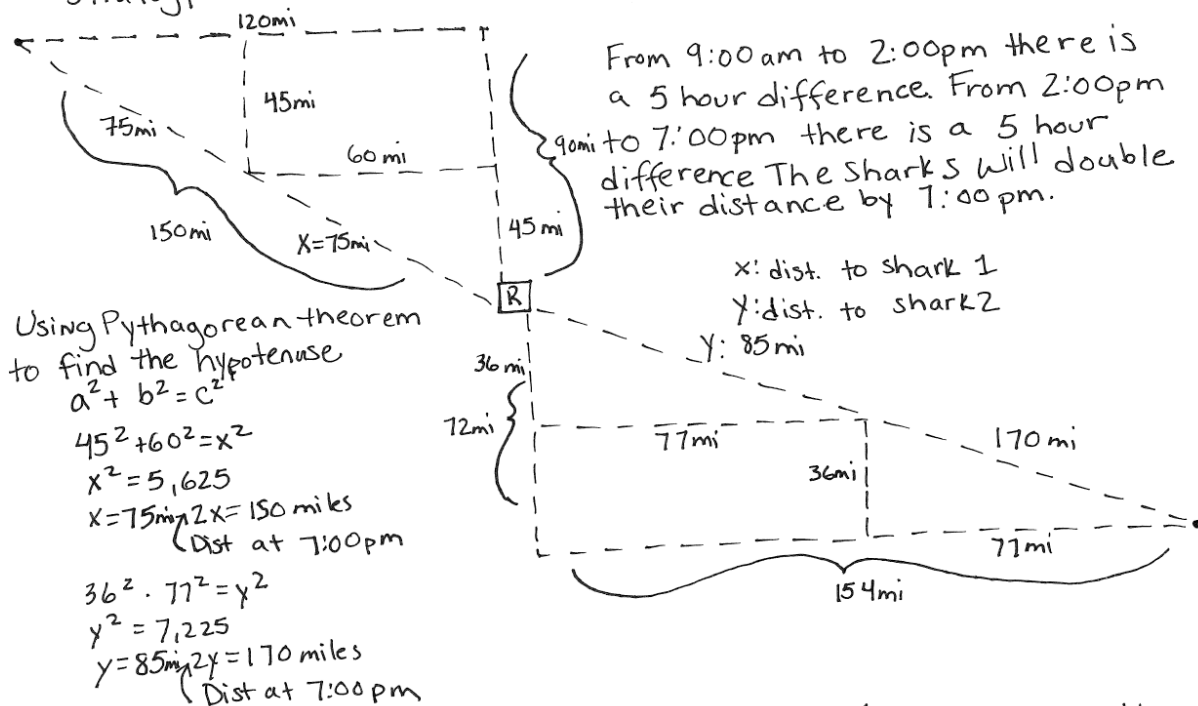
P/S	R/P	Com	Con	Rep	A/Level
E	E	E	E	E	E

### Scary Reunion

A team of marine biologists are studying sharks, how fast would the team's research vessel need to travel in order to catch a shark by 7:00 pm if they leave at 2:00 pm?

I am going to make a diagram, as this will help me visualize the shark locations and come up with a solution.

#### Strategy 1:



To find the necessary speed, I need to divide distance (150 mi, 170 mi) by time (5 hours)

Shark 1:  $\frac{150}{5} = 30$  30 miles per hour to catch the shark

Shark 2:  $\frac{170}{5} = 34$  34 miles per hour to catch the shark.

Speed of Shark 1 =  $\frac{75}{5} = 15 \text{ mph}$   
 $15 \cdot 2 = 30$  Boat travels 30 mph  
 Speed of Shark 2 =  $\frac{85}{5} = 17 \text{ mph}$   
 $17 \cdot 2 = 34$  Boat travels 34 mph

The boat has to travel at double the speed of the sharks, because it travels double the distance in the same amount of time.



## Expert Page 2

### Scary Reunion Explanation

I started solving this problem by making a diagram that represents the location of the sharks in relation to the research boat. Shark 1 was 45 miles north and 60 miles west, and Shark 2 was 36 miles south and 77 miles east. Because the shark's travel pattern creates a right triangle, I was able to use the Pythagorean theorem to find the hypotenuses of the two triangles. Shark 1 was 75 miles away and Shark 2 was 85 miles away. At 9:00 am, the sharks were released and traveled their respective distances by 2:00 pm (a 5 hour difference). By 7:00 pm (when the researchers have to catch the shark by), the sharks will have doubled their distance so the researchers have to travel 150 miles for shark 1 and 170 miles for shark 2 (both in 5 hours). Dividing distance by time, I get the necessary speed for shark 1 to be 30 miles per hour and the necessary speed for shark 2 to be 34 miles per hour. Another way I did it was to compare the speed of the sharks to the boat speed. Shark 1 travels 75 miles in 5 hours, so its speed is 15 mph. The boat has to be double that, because it travels double the distance in the same time. Therefore, the necessary speed for the boat for shark 1 again turns out to be 30 mph. For shark 2, its speed is 17 mph ( $\frac{85}{5}$ ), so the boat's speed has to be 34 mph.