

## **WARNING!**

The resource you are about to see may LOOK like an ordinary  
**word problem or story problem.**

## **IT'S NOT**

**Exemplars are performance tasks... and they're far superior.**

### **Exemplars are POWERFUL**

They're real-world problems that KIDS get to decide how to solve. Our tasks are engineered to be solved in many different ways, so everyone can jump in and find a strategy that works for them. No hand-holding here. Kids get to think critically, be creative, and apply their math skills to authentic situations.

### **Exemplars are HIGHLY POTENT**

In the real world, math is everywhere! When solving Exemplars tasks, kids exercise their ENTIRE brains by practicing things like:



### **Math Solutions are THE KEY**

Are your students answer-getters? Not with Exemplars. Developing a mathematical solution is what builds lasting memories and a DEEP UNDERSTANDING of math concepts. And we'll teach you how to get there!

**Ready to build confidence, unlock your students' inner mathematicians, and celebrate all those 'aha' moments?**

**Let's Go!**

## A Guide to Exemplars Resources

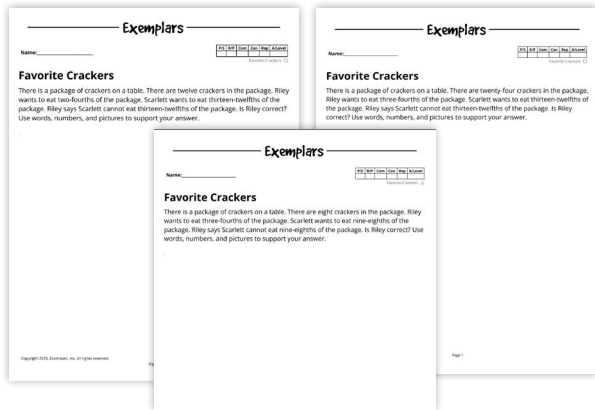
Exemplars problem-solving performance tasks are thoughtfully written and classroom-tested. Our rich tasks may be used for assessment, instruction, professional development, or to build a thinking classroom. Exemplars is the perfect supplement to your curriculum!

### Tasks Include:

#### Differentiated Versions

Standard Version

More Challenging Version



More Accessible Version

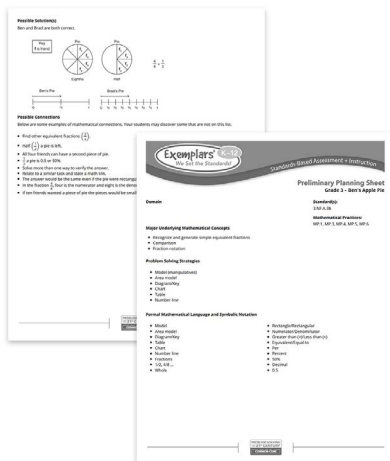
#### Engagement Images (to pique student curiosity)



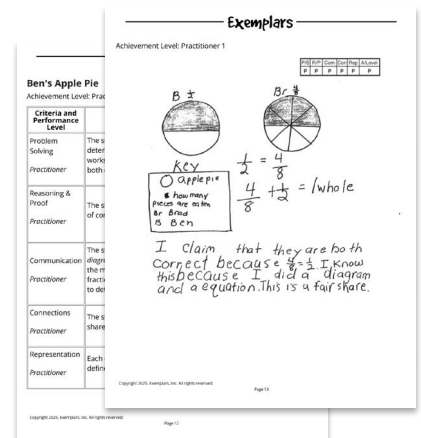
#### Standards-Based Math Rubric

Standards-Based Math Rubric	
Practitioner	Expert
<p><b>Problem Solving</b></p> <p>Identifies the problem and the information given. Plans a strategy to solve the problem. Uses a variety of strategies to solve the problem. Checks the solution to make sure it is correct.</p>	<p><b>Problem Solving</b></p> <p>Identifies the problem and the information given. Plans a strategy to solve the problem. Uses a variety of strategies to solve the problem. Checks the solution to make sure it is correct. Explains the solution to others.</p>
<p><b>Reasoning &amp; Proof</b></p> <p>Explains the solution to others. Uses a variety of strategies to solve the problem. Checks the solution to make sure it is correct.</p>	<p><b>Reasoning &amp; Proof</b></p> <p>Explains the solution to others. Uses a variety of strategies to solve the problem. Checks the solution to make sure it is correct. Explains the solution to others.</p>
<p><b>Communication</b></p> <p>Explains the solution to others. Uses a variety of strategies to solve the problem. Checks the solution to make sure it is correct.</p>	<p><b>Communication</b></p> <p>Explains the solution to others. Uses a variety of strategies to solve the problem. Checks the solution to make sure it is correct. Explains the solution to others.</p>
<p><b>Connections</b></p> <p>Explains the solution to others. Uses a variety of strategies to solve the problem. Checks the solution to make sure it is correct.</p>	<p><b>Connections</b></p> <p>Explains the solution to others. Uses a variety of strategies to solve the problem. Checks the solution to make sure it is correct. Explains the solution to others.</p>
<p><b>Representation</b></p> <p>Explains the solution to others. Uses a variety of strategies to solve the problem. Checks the solution to make sure it is correct.</p>	<p><b>Representation</b></p> <p>Explains the solution to others. Uses a variety of strategies to solve the problem. Checks the solution to make sure it is correct. Explains the solution to others.</p>

#### Lesson Planning Sheets and Possible Solutions



#### Scored Student Work Samples (examples of math solutions at 4 performance levels)



## Engagement Image to Launch Task

*Teachers use this resource to pique student curiosity.*



## Rides at Fun Park

Twelve friends are at Fun Park. There are many rides at the park. The Tilty Whirl has cars that hold two people each. The Ferris Wheel has cars that hold four people each. The Roller Coaster has cars that hold six people each. The Water Log has cars that hold five people each. All cars must be filled with people before a ride starts. The friends do not want to share a ride with anyone they do not know. Which rides can the friends ride? Show all of your mathematical thinking.

## Rides at Fun Park

### Common Core Task Alignments

**Mathematical Practices:** 1, 3, 4, 5, 6

**Grade 3 Content Standards:**  
3.OA.A.3

### Task

Twelve friends are at Fun Park. There are many rides at the park. The Tilty Whirl has cars that hold two people each. The Ferris Wheel has cars that hold four people each. The Roller Coaster has cars that hold six people each. The Water Log has cars that hold five people each. All cars must be filled with people before a ride starts. The friends do not want to share a ride with anyone they do not know. Which rides can the friends ride? Show all of your mathematical thinking.

### Alternative Versions of the Task

#### **More Accessible Version:**

Twelve friends are at Fun Park. There are many rides at the park. The Tilty Whirl has cars that hold two people each. The Ferris Wheel has cars that hold four people each. The Water Log has cars that hold five people each. All cars must be filled with people before a ride starts. The friends do not want to share a ride with anyone they do not know. Which rides can the friends ride? Show all of your mathematical thinking.

#### **More Challenging Version:**

Eighteen friends are at Fun Park. There are many rides at the park. The Tilty Whirl has cars that hold two people each. The Ferris Wheel has cars that hold four people each. The Roller Coaster has cars that hold six people each. The Water Log has cars that hold five people each. All cars must be filled with people before a ride starts. The friends do not want to share a ride with anyone they do not know. Which rides can the friends ride? Show all of your mathematical thinking.

## Common Core Content Standards and Evidence

### 3.OA Operations and Algebraic Thinking

*Represent and solve problems involving multiplication and division.*

3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

### Exemplars Task-Specific Evidence

This task requires students to understand the meaning of division. Students will need to partition the total (12) into fair shares and use the factors of 12 to recognize when the total cannot be partitioned into equal groups of five without having a remainder.

## **Underlying Mathematical Concepts**

- Using multiplication/division within 100
- Partitioning into equal shares
- Factors of 12
- Finding a missing factor when the product and one factor are known
- Repeated subtraction of equal parts

## **Possible Problem-Solving Strategies**

- Model (manipulatives)
- Diagram/Key
- Array
- Chart
- Number line

## **Possible Mathematical Vocabulary/Symbolic Representation**

- Model
- Diagram/Key
- Chart
- Number line
- Equivalent to
- Equal sets/Equal parts/Equal shares
- Sets/Groups
- Array
- Total/Sum
- Amount
- Most/Least
- Odd/Even
- Equation
- Quotient
- Divisor
- Dividend
- Product
- Factor
- Remainder
- Per
- Dozen

## Possible Solutions

### Original Version:

The 12 friends can ride the Tilty Whirl, Roller Coaster and the Ferris Wheel.

Ride	People per Car	Total Cars Filled	Remaining People
Tilty Whirl	2	6	0
Ferris Wheel	4	3	0
Roller Coaster	6	2	0
Water Log	5	2	2

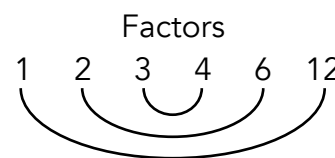
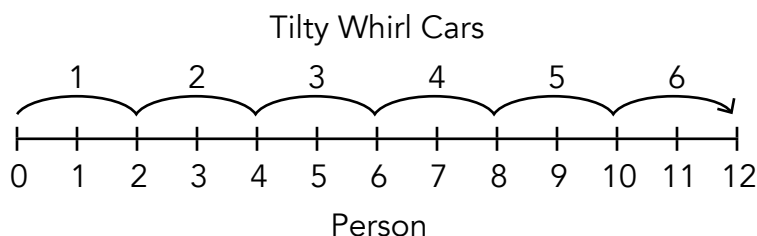
$$2 \times 6 = 12$$

$$12 \div 4 = 3$$

$$12 - 6 = 6$$

$$6 - 6 = 0$$

$$5 + 5 = 10 \quad 2 \text{ remaining people}$$



### More Accessible Version:

The 12 friends can ride the Tilty Whirl and the Ferris Wheel.

### More Challenging Version:

The 18 friends can ride the Tilty Whirl and the Roller Coaster.

## Possible Connections

Below are some examples of mathematical connections. Your students may discover some that are not on this list.

- 12 is a dozen friends.
- There are an even number of friends.
- There are an even number of friends in each car except for the Water Log, which holds an odd number.
- The Roller Coaster uses the least amount of cars.
- The Tilty Whirl uses the most cars.
- Multiplication is used to support addition.
- 1, 2, 3, 4, 6 and 12 are called factors of 12.
- Solve more than one way to verify the answer.
- Relate to another task and state a math link.



## Engagement Image to Launch Task

*Teachers use this resource to pique student curiosity.*





## Three Fish Limit

Some friends are camping near the shore. One friend suggests that they go fishing. Six boats are available at the campsite. Each boat holds two people and the friends are excited because that is exactly how many they need. The rule on this lake is that each person can catch only three fish. How many fish did the friends catch that day if each friend caught the limit? Show all your mathematical thinking.

## Three Fish Limit

### Common Core Task Alignments

**Mathematical Practices:** 1, 3, 4, 5, 6, 7

**Grade 3 Content Standards:**  
3.OA.A.3

### Task

Some friends are camping near the shore. One friend suggests that they go fishing. Six boats are available at the campsite. Each boat holds two people and the friends are excited because that is exactly how many they need. The rule on this lake is that each person can catch only three fish. How many fish did the friends catch that day if each friend caught the limit? Show all your mathematical thinking.

### Common Core Content Standards and Evidence

#### 3.OA Operations and Algebraic Thinking

*Represent and solve problems involving multiplication and division.*

3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

#### Exemplars Task-Specific Evidence

This task requires students to use repeated addition or multiplication to find the number of people who are camping and the number of fish they catch.

#### Underlying Mathematical Concepts

- 2 to 1
- 3 to 1
- Addition/Counting on
- Multiplication within 100
- Number sense to 36
- Combining

#### Possible Problem-Solving Strategies

- Model (manipulatives)
- Diagram/Key
- Table
- Tally chart
- Number line

## Possible Mathematical Vocabulary/Symbolic Representation

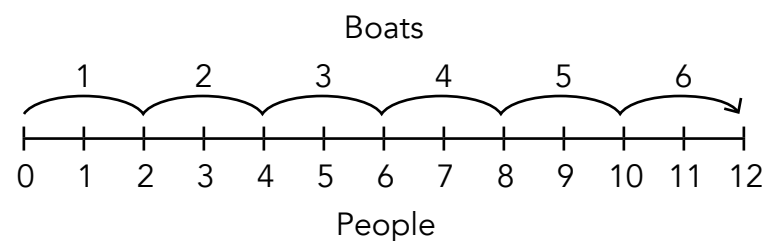
- Model
- Diagram/Key
- Table
- Tally chart
- Number line
- Pattern
- Multiples
- Per
- Odd/Even
- More/Less than
- $\frac{1}{2}$
- Rule
- $2 \cdot b = p$  ( $b + b = p$ ),  $6 \cdot b = f$
- Variables
- Total
- Product
- Dozen
- Pair
- Input/Output
- Array
- Graph
- Axis
- Sets
- Groups

## Possible Solutions

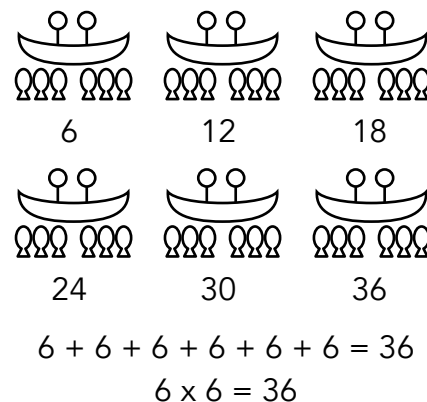
The friends caught 36 fish.

Boat	People	Fish
1	2	6
2	4	12
3	6	18
4	8	24
5	10	30
6	12	36

Rule
<b>b</b> is boat
<b>p</b> is people
<b>f</b> is fish
$2 \cdot b = p$
$3 \cdot p = f$
$6 \cdot b = f$



Key
is 1 Person
is 1 Fish
is 1 Boat



Boat	1	2	3	4	5	6
People						
Fish						

$$2 \times 6 = 12$$

$$3 \times 12 = 36$$

$$6 \times 6 = 36$$

## Possible Connections

Below are some examples of mathematical connections. Your students may discover some that are not on this list.

- Patterns: Boat +1, People +2, Fish +6.
- The people and amount of fish caught are even numbers.
- 1 person =  $\frac{1}{2}$  boat.
- 2 people is a pair.
- Multiples of 2 and 6 are even numbers.
- There are 4 more fish than people.
- Graph input/outputs on graph paper.
- Rules:  $2 \cdot b = p$ ,  $6 \cdot b = f$  (b is boat, p is person, f is fish)
- Relate to a similar task and state a math link.
- Solve more than one way to verify the answer.

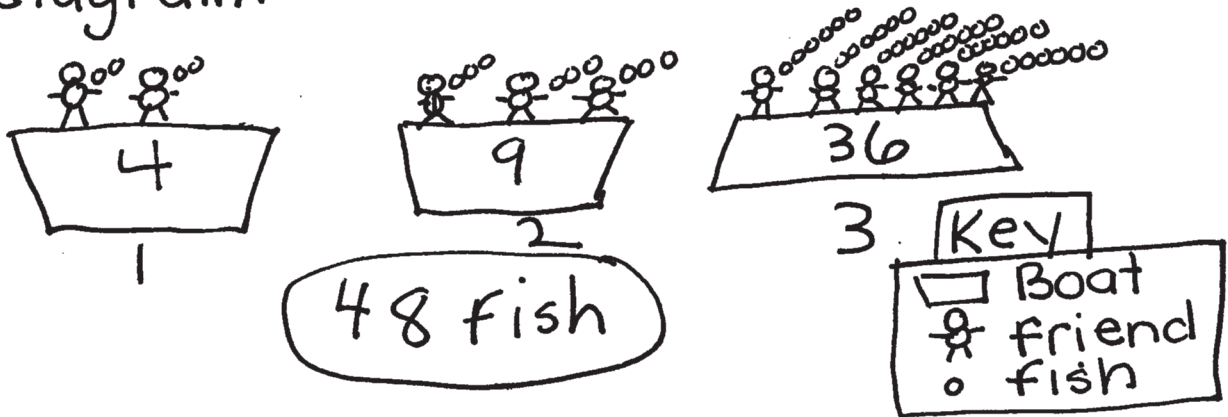
# Novice Scoring Rationales

Criteria and Performance Level	Assessment Rationales
<b>Problem Solving</b> <i>Novice</i>	The student's strategy of using a diagram to show three boats with two friends in a boat catching two fish each, three friends in a boat catching three fish and six friends in a boat catching six fish would not work to solve this task. The student's answer, "48 fish," is not correct.
<b>Reasoning Proof</b> <i>Novice</i>	The student does not show any correct conceptual understanding of the problem. It appears that the student confuses three fish with three boats and then has the friends catch a number of fish equal to the number of friends in each boat.
<b>Communication</b> <i>Practitioner</i>	The student correctly uses the mathematical terms <i>diagram</i> and <i>key</i> . Although the diagram represents incorrect data, the use of the communication terms is correct.
<b>Connections</b> <i>Novice</i>	The student does not make a mathematically relevant observation about her/his solution.
<b>Representation</b> <i>Apprentice</i>	The use of a diagram is appropriate to this task but the student's work is not correct. The student does not represent six boats with two friends in each boat and each friend catching three fish.

# Novice

P/S	R/P	Com	Con	Rep	A/Level
N	N	P	N	A	N

I have to find fish. I will make a diagram.



Boat 3 caught the most fish.  
The fish pattern is 2 fish, 3 fish,  
6 fish.



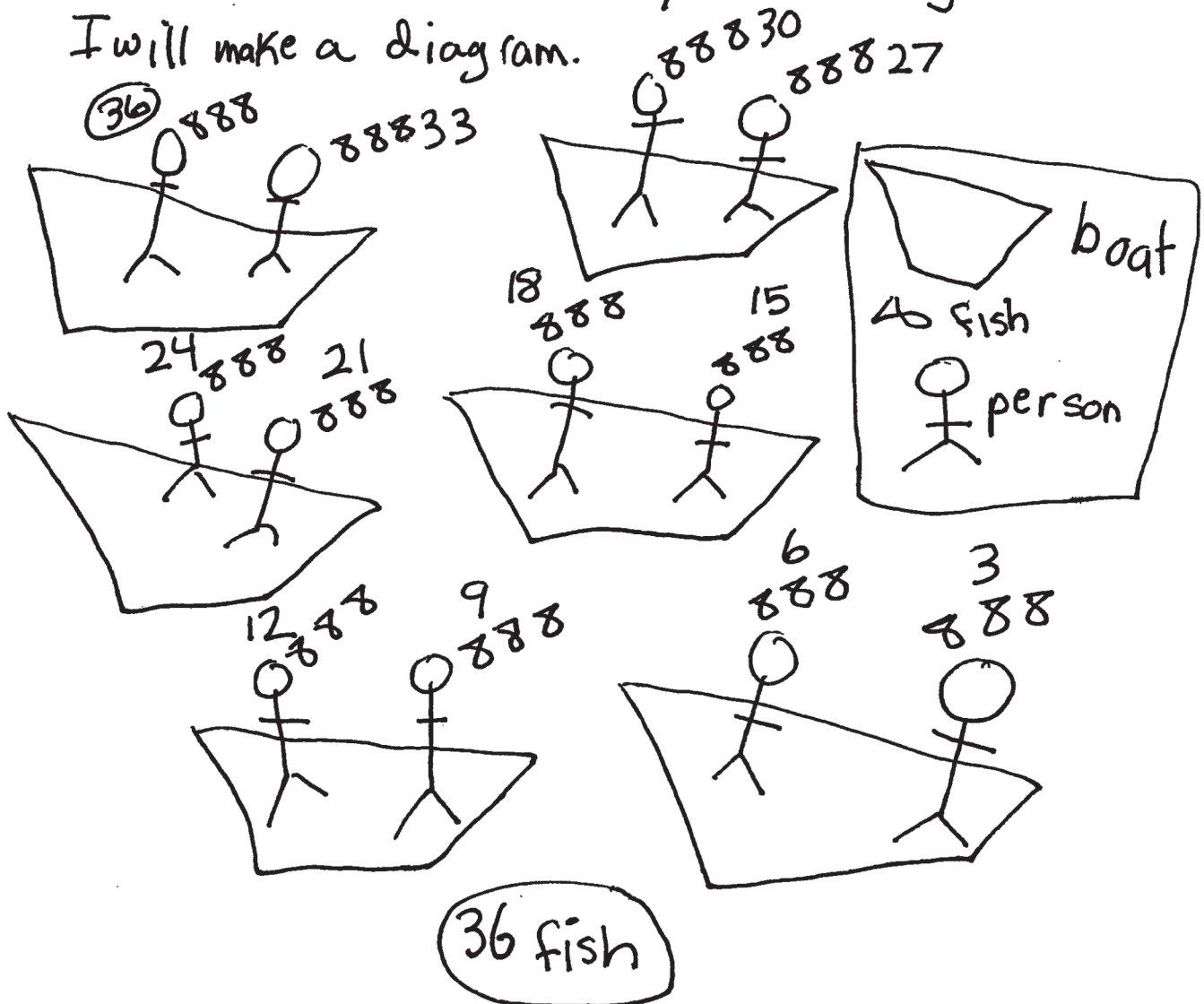
# Apprentice Scoring Rationales, Student 1

Criteria and Performance Level	Assessment Rationales
<b>Problem Solving</b> <i>Practitioner</i>	The student's strategy of using a diagram and a key to represent the six boats, two friends per boat, and three fish per friend would work to solve this task. The student states a correct answer, "36 fish."
<b>Reasoning Proof</b> <i>Practitioner</i>	The student's arguments are constructed with adequate mathematical basis. The student's solution is systematic and supports correct reasoning.
<b>Communication</b> <i>Apprentice</i>	The student correctly uses the mathematical term <i>diagram</i> .
<b>Connections</b> <i>Novice</i>	The student does not make a mathematically relevant observation about her/his solution.
<b>Representation</b> <i>Practitioner</i>	The student's diagram is appropriate and accurate. The student provides labels in a key for a boat, fish, and person. The student does not have to use the term key, but it would have provided another mathematical term to the student's solution.

# Apprentice, Student 1

P/S	R/P	Com	Con	Rep	A/Level
P	P	A	N	P	A

I need to find out how many fish caught.  
I will make a diagram.



# Apprentice Scoring Rationales, Student 2

Criteria and Performance Level	Assessment Rationales
<b>Problem Solving</b> <i>Apprentice</i>	The student's strategy of making a table for boats and fish would work to solve the task but the fish column only represents the fish caught by one friend per boat and not two friends per boat. The student's answer, "The answer is 18 fish," is incorrect.
<b>Reasoning Proof</b> <i>Apprentice</i>	The student has a flaw in her/his reasoning. The running total of fish in the second column in the student's table only represents three fish per friend and not six fish for two friends.
<b>Communication</b> <i>Apprentice</i>	The student correctly uses the mathematical term <i>table</i> .
<b>Connections</b> <i>Novice</i>	The student does not make a mathematically relevant observation about her/his solution.
<b>Representation</b> <i>Apprentice</i>	The student's table is appropriate but not accurate. The fish column has incorrect data. The data should represent 6, 12, 18, 24, 30, 36 fish.

## Apprentice, Student 2

P/S	R/P	Com	Con	Rep	A/Level
A	A	A	N	A	A

I have to find how many fish they caught on the lake to solve the problem. I will use a table.

boats	Fish
1	3
2	6
3	9
4	12
5	15
6	18

The answer is  
18 Fish

I figured my answer on my table. There are 18 fish.

# Practitioner Scoring Rationales, Student 1

Criteria and Performance Level	Assessment Rationales
<b>Problem Solving</b> <i>Practitioner</i>	The student's strategy of making a table to show six boats, two people, and a running total of fish caught would work to solve the task. The student states a correct answer, "They caught a total of 36 fishes."
<b>Reasoning Proof</b> <i>Practitioner</i>	The student provides adequate evidence of her/his reasoning both in her/his table and text. The student applies understanding of multiplication by applying the counting by one, two, and six patterns.
<b>Communication</b> <i>Practitioner</i>	The student correctly uses the mathematical terms <i>table</i> , <i>total</i> and <i>pattern</i> .
<b>Connections</b> <i>Practitioner</i>	The student makes the mathematically relevant observations, "The patterns are boats +1, friends + 2, fish caught +6."
<b>Representation</b> <i>Practitioner</i>	The student's table is appropriate and accurate. All necessary labels are provided and the data is correct.

# Practitioner, Student 1

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

I will find out how many fish they catch altogether. I will use a table to solve this Problem

boats	friends	fish caught
1	2	6
2	4	12
3	6	18
4	8	24
5	10	30
6	12	36

They caught a total of 36 fishes

The patterns are  
boat + 1 friend + 2  
fish caught + 6



# Practitioner Scoring Rationales, Student 2

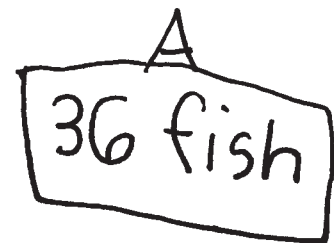
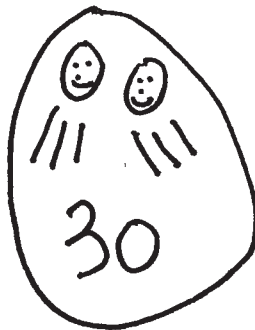
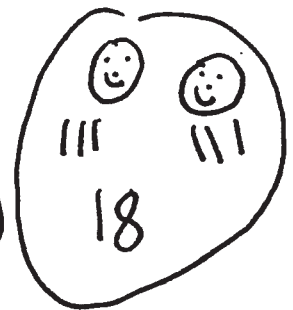
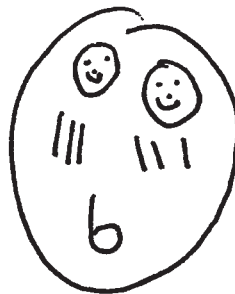
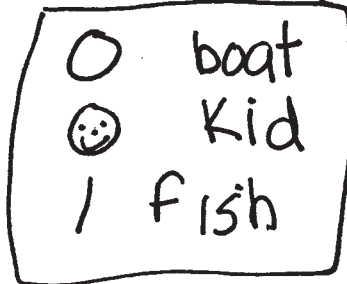
Criteria and Performance Level	Assessment Rationales
<b>Problem Solving</b> <i>Practitioner</i>	The student's strategy of diagramming the boats, kids, and fish, and providing an accurate key works to solve the task. The student states a correct answer, "36 fish."
<b>Reasoning Proof</b> <i>Practitioner</i>	The student demonstrates understanding of the mathematical concept of applying multiplication and patterns to find the total number of fish that can be caught while fishing. The student also applies multiplication and addition in determining the total cost of the 36 fish sold at 50 cents per fish.
<b>Communication</b> <i>Practitioner</i>	The student correctly uses the mathematical terms <i>diagram</i> , <i>key</i> , <i>pattern</i> , <i>dozen</i> and <i>cost</i> . The student correctly uses the mathematical notation \$.50, \$3.00, \$9.00, \$18.00.
<b>Connections</b> <i>Practitioner</i>	The student makes the mathematically relevant observations, "I saw the add 6 pattern for fish," and, "I saw that there were 12 kids and that is a dozen." The student adds her/his own part two of the task by determining that if each fish sold for \$.50, the total cost would be \$18.00.
<b>Representation</b> <i>Practitioner</i>	The student's diagram is appropriate and accurate. A key is used to represent the boats, kids and fish.

## Practitioner, Student 2

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

I need to find out how many fish.  
I will make a diagram.

Key



I saw the add 6 pattern  
I saw That there were 12 Kids  
and that is a dozen

## Practitioner, Student 2 (cont.)

If they sold the fish for \$.50

$$\begin{array}{l}
 \$.50 \times 6 = \$3.00 \\
 \$.50 \times 6 = \$3.00 \\
 \$.50 \times 6 = \$3.00 \\
 \$.50 \times 6 = \$3.00 \\
 \$.50 \times 6 = \$3.00 \\
 \$.50 \times 6 = \$3.00
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 \left. \begin{array}{l} \\ \\ \end{array} \right\} \$9.00
 \end{array}
 \begin{array}{l}
 \\ \\
 \end{array}
 \left. \begin{array}{l} \$9.00 \\ \$9.00 \end{array} \right\} \$18.00$$

It would cost \$18.00

# Practitioner Scoring Rationales, Student 3

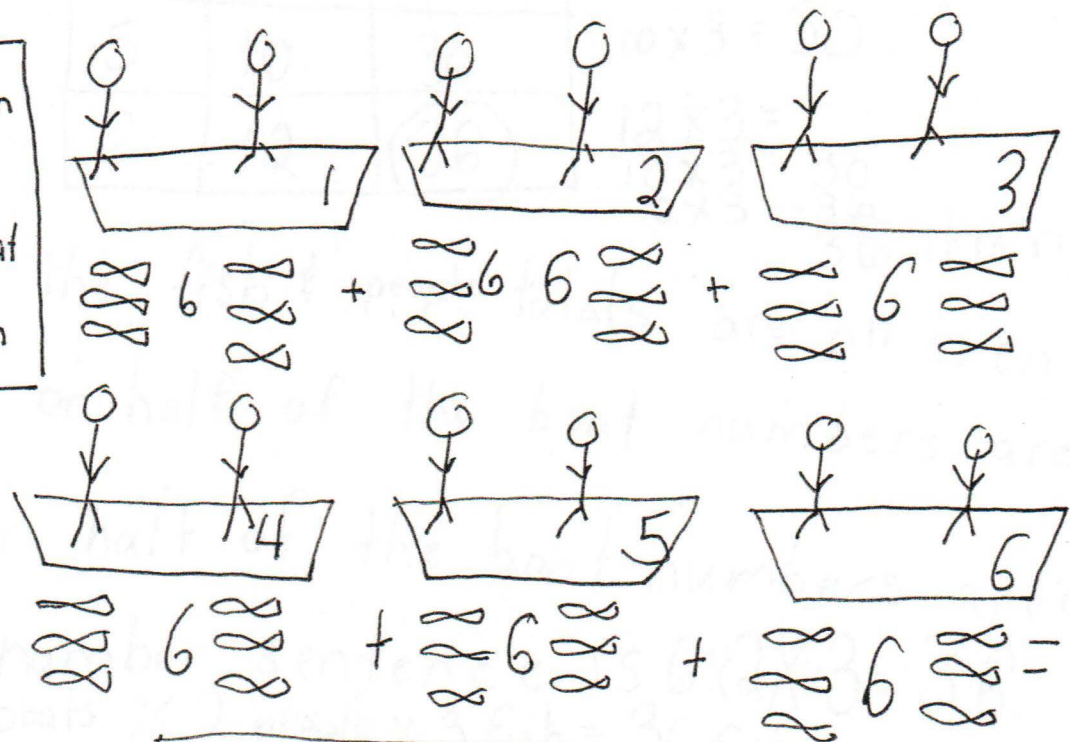
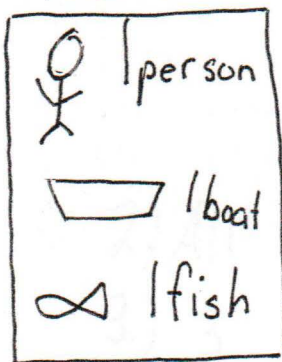
Criteria and Performance Level	Assessment Rationales
<b>Problem Solving</b> <i>Expert</i>	The student's strategy of diagramming the persons, boats, and fish, and providing an accurate key works to solve the task. The student states a correct answer, "36 fish in all." The student verifies her/his answer by using a table to compare with the data in her/his diagram.
<b>Reasoning Proof</b> <i>Expert</i>	The student demonstrates understanding of the mathematical concept of applying multiplication patterns to find the total number of fish that can be caught while fishing. The student also applies multiplication and addition in a table to verify her/his answer. The student brings the understanding of fractions to the task.
<b>Communication</b> <i>Practitioner</i>	The student correctly uses the mathematical terms <i>total</i> , <i>diagram</i> , <i>key</i> , <i>table</i> , <i>even</i> , <i>odd</i> , <i>half</i> , <i>number sentence</i> , <i>dozen</i> , <i>sum</i> . The student correctly uses the mathematical notation $\frac{3}{6}$ . All of these terms are considered typical to the Grade three Common Core standards and do not earn a performance level of Expert.
<b>Connections</b> <i>Expert</i>	The student makes the mathematically relevant Practitioner connections, "All the fish & people totals are all even," and, "A number sentence is $6 \times 2 \times 3 = 36$ . 6 boats $\times$ 2 people $\times$ 3 fish = 36 fish." The student makes the Expert connection of applying an understanding of fractions to the task. The student states, " $\frac{3}{6}$ or half of the boat numbers are odd," " $\frac{3}{6}$ or half of the boat numbers are even," and "The boat sum is half of the people sum." The student verifies her/his answer by comparing the data on her/his diagram with the data on a table and stating, "it is right."
<b>Representation</b> <i>Expert</i>	The student's diagram is appropriate and accurate. A key is used to represent the persons, boats, and fish. The student's table is appropriate and accurate. All labels are provided and the the entered data is correct. The student compares the data of her/his diagram and table to help verify that the answer is correct.

## Practitioner, Student 3

P/S	R/P	Com	Con	Rep	A/Level
E	E	P	E	E	P

I need to find out how many fish they caught total. I will make a diagram.

KEY



Answer: 36 fish in all.

## Practitioner, Student 3 (cont.)

### Connections

1.) I will prove my answer another way using a table.

total boat	total people	total fish
1	2	6
2	4	12
3	6	18
4	8	24
5	10	30
6	12	(36)

$$2 \times 3 = 6$$

$$4 \times 3 = 12$$

$$6 \times 3 = 18$$

$$8 \times 3 = 24$$

$$10 \times 3 = 30$$

$$12 \times 3 = 36$$

$$10 \times 3 = 30$$

$$2 \times 3 = 6$$

36 it is right.

2.) All the fish & people totals are all even.

3.)  $\frac{3}{6}$  or half of the boat numbers are odd.

4.)  $\frac{3}{6}$  or half of the boat numbers are even.

5.) A number sentence is  $6 \times 2 \times 3 = 36$ .

6 boats  $\times$  2 people  $\times$  3 fish = 36 fish

6.) There is 1 dozen of people in all.

7.) The boat sum is half of the people sum.



# Expert Scoring Rationales

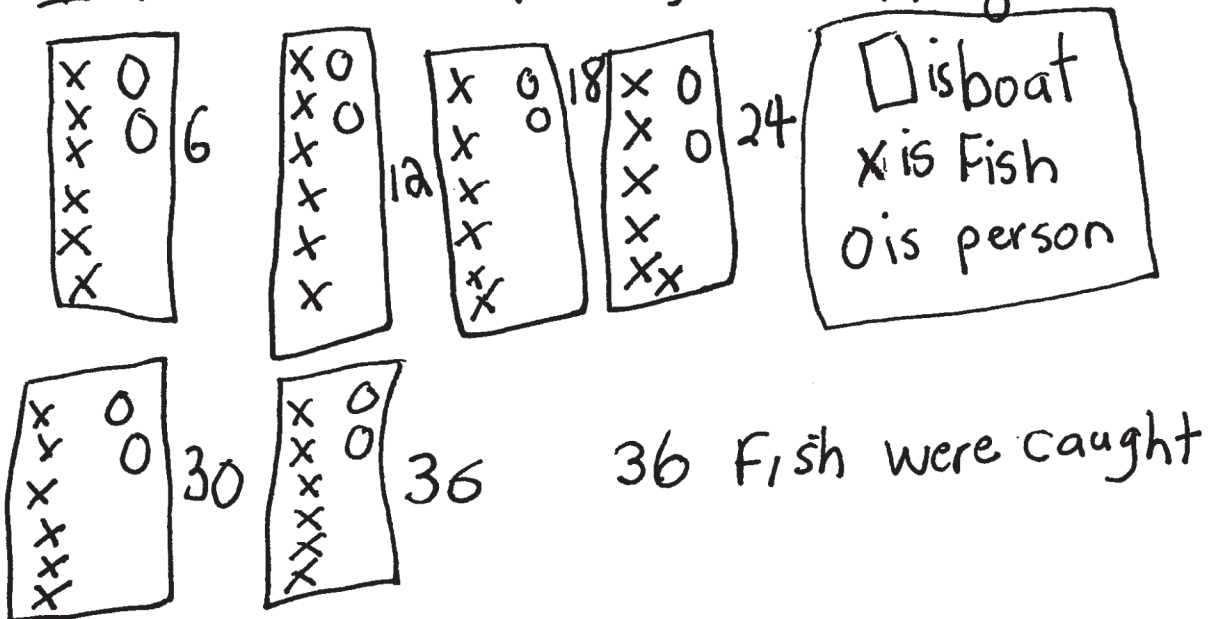
Criteria and Performance Level	Assessment Rationales
<b>Problem Solving</b> <i>Expert</i>	The student's strategy of using a diagram to determine the total number of fish caught by 12 people works to solve the task. The student's answer, "36 fish were caught," is correct. The student verifies her/his answer by making a table and equations. The student also verifies her/his answer by generalizing a rule for finding the total number of fish caught for any number of boats.
<b>Reasoning Proof</b> <i>Expert</i>	The student demonstrates correct reasoning of the underlying concepts in the problem. The student uses multiplication correctly. The student generalizes a rule, $6 \cdot b = f$ , to verify that the data in her/his table is accurate, the answer is correct, and to find the number of fish for larger numbers of boats.
<b>Communication</b> <i>Expert</i>	The student correctly uses the mathematical terms <i>diagram</i> , <i>key</i> , <i>table</i> , <i>total</i> , <i>more than</i> , <i>dozen</i> , <i>pattern</i> and <i>rule</i> . The student correctly uses symbolic notation $6 \cdot b = f$ with the variables "b" and "f" defined in a key.
<b>Connections</b> <i>Expert</i>	The student makes the Practitioner mathematically relevant observations, "The boat pattern is + 1," "The friends pattern is + 2," "The fish pattern is + 6," "There are more total fish than boats or friends," "36 fish is 3 dozen." The student makes the Expert connection of generalizing a rule, $6 \cdot b = f$ . The student supplies a key to define the variables, b and f, and states, "Each boat will always have two people in it." The student uses the rule to verify that the data in her/his table is accurate and states, "I am right because I get 36 fish all the time." The student also uses the rule to find the number of fish for 10 and 50 boats.
<b>Representation</b> <i>Expert</i>	The student's diagram is appropriate to the problem and accurate. A key is provided to define boat, fish, and person. The student's table is appropriate and accurate with all necessary labels and correctly entered data. The student uses her/his table to analyze the relationship between the boats and fish to generalize a rule to find the total number of fish for any number of boats.

# Expert

P/S	R/P	Com	Con	Rep	A/Level
E	E	E	E	E	E

I need to find all the fish caught.

I will make a diagram. Key



## Expert (cont.)

Fishing table

boat	friends	total fish
1	2	6
2	4	12
3	6	18
4	8	24
5	10	30
6	12	36

my connections  
The boat pattern is +1. The friends pattern is +2.  
The fish pattern is +6

There are more total fish than

I am right because I get 36 fish all the time. boats or friends  
36 fish is 3 dozen

key

b boat  
f fish

Each boat will always have two people in it

rule

$$6 \cdot b = f$$

$$6 \cdot 3 = 18$$

$$6 \cdot 6 = 36$$

$$6 \cdot 10 = 60$$

$$6 \cdot 50 = 300$$

I get 36 fish again.