Grade 7 Sample Instructional Math Task

Instructional

Engagement Image to Launch Task

Teachers use this resource to pique student curiosity.

Dragon Races

The 479th Annual Dragon Race is today on the Island of Barnacle. The race is out to Otter Island and back. The three fastest dragons, Manic, Crater, and Comet, have been invited to compete.

After $\frac{1}{3}$ of an hour Manic is seen $\frac{1}{4}$ of the way to Otter Island. Crater is seen turning around at Otter Island after 45 minutes. Comet is seen returning from Otter Island halfway back to the Island of Barnacle after $1\frac{1}{3}$ hours.

Based on the different sightings of the dragons, who do you predict will return to the finish line first? Provide mathematical evidence to support your prediction.

Dragon Races

The 479th Annual Dragon Race is today on the Island of Barnacle. The race is out to Otter Island and back. The three fastest dragons, Manic, Crater, and Comet, have been invited to compete.

After $\frac{1}{3}$ of an hour Manic is seen $\frac{1}{4}$ of the way to Otter Island. Crater is seen turning around at Otter Island after 45 minutes. Comet is seen returning from Otter Island halfway back to the Island of Barnacle after $1\frac{1}{3}$ hours.

Based on the different sightings of the dragons, who do you predict will return to the finish line first? Provide mathematical evidence to support your prediction.

Alternative Versions of the Task

More Accessible Version

The 479th Annual Dragon Race is today on the Island of Barnacle. The three fastest dragons have been invited to compete. The race is out to Otter Island and back.

After $\frac{1}{3}$ of an hour Manic is seen $\frac{1}{4}$ of the way to Otter Island.

After $\frac{3}{4}$ of an hour Crater is seen turning around at Otter Island.

After $1\frac{1}{2}$ hours Comet is seen returning from Otter Island halfway back to the Island of Barnacle.

Based on the different sightings of the dragons, who do you predict will return home first? Provide mathematical evidence to support your prediction.

More Challenging Version

The 479th Annual Dragon Race is today on the Island of Barnacle. The race is out to Otter Island and back. The three fastest dragons have been invited to compete. Based on the dragons measurements, provided below, which would you predict to be the fastest? Why?

Manic is 20 feet tall, with a wingspan of 40 feet and weighs 720 pounds. Crater is 45 feet tall, with a wingspan of 90 feet and weighs 3,780 pounds. Comet is 8 feet tall, with a wingspan of 16 feet and weighs 352 pounds.

Once the race is underway, officials report out the following information.

After $\frac{1}{3}$ of an hour Manic is seen $\frac{3}{7}$ of the way to Otter Island. Crater is seen turning around at Otter Island after 45 minutes. Comet is finally seen halfway back to the Island of Barnacle after $1\frac{1}{9}$ hours.

Based on the different sightings of the dragons, who do you predict will return home first? How does this compare to your original prediction? Provide mathematical evidence to support your thinking.

Planning Sheet

Dragon Races

Common Core Task Alignments

Mathematical Practices: MP.1 MP.2 MP.3 MP.4 MP.5

Grade 7 Content Standards: 7.RP.A.1

Common Core Standards and Evidence

7.RP.A.1

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour with 2 being the unit rate.

Exemplars Task-Specific Evidence

This task requires students to find and compare rates in order to determine which of three dragons will be the first to finish a race between two islands and back.

Underlying Mathematical Concepts

- Reasoning about ratios and rates
- Finding unit rates
- Comparing and operations with rational numbers

Possible Problem-Solving Strategies

- Find and compare unit rate
- Equivalent ratios
- Ratio table
- Double number line
- Graph

Possible Mathematical Vocabulary/Symbolic Representation

- Complex fraction
- Unit rate
- Quotient
- Scaling
- Scale factor
- Proportional relationship
- Rate
- Ratio
- Equivalent ratio
- Independent variable
- Dependent variable
- Coordinate point
- Rational number
- Benchmark
- Greater than (>)
- Less than (<)

Instructional

Possible Solutions

Crater will return to the finish line first based on where each dragon was seen and assuming they continue the race at that same rate.

Students may decide to use the distance from the Island of Barnacle to Otter Island as the unit or the whole race as the unit.

Where each dragon was seen:

Manic was seen $\frac{1}{4}$ of the way to Otter Island which is $\frac{1}{8}$ of the race.
Crater was seen at Otter Island which is $\frac{1}{2}$ of the race.
Comet was seen returning from Otter Island halfway back to the Island of Barnacle which is $\frac{3}{4}$ of the race.

Find and Compare Unit Rate (based on whole race distance as the unit)

Manic:

$\frac{\frac{1}{8}}{\frac{1}{3}} = \frac{1}{8} \div \frac{1}{3} = \frac{1}{8} \times 3 = \frac{3}{8}$ *of the race per hour*

Manic's speed is less than $\frac{4}{8}$ or $\frac{1}{2}$.

Crater: 45 minutes = $\frac{3}{4}$ of an hour

$\frac{\frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6}$ of the race per hour

Crater's speed is greater than $\frac{3}{6}$ or $\frac{1}{2}$ by $\frac{1}{6}$ (or 0.1666...) which means Crater is faster than Manic.

Comet:

$\frac{\frac{6}{8}}{\frac{4}{3}} = \frac{6}{8} \div \frac{4}{3} = \frac{6}{8} \times \frac{3}{4} = \frac{18}{32}$ of the race per hour

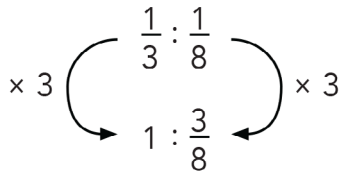
Comet's speed is greater than $\frac{16}{32}$ or $\frac{1}{2}$ by $\frac{2}{32}$ or $\frac{1}{16}$ (or 0.0625).

Comet is also faster than Manic, but slower than Crater, since Comet is a smaller amount greater than $\frac{1}{2}$.

Crater is traveling the fastest and will return to the finish line first.

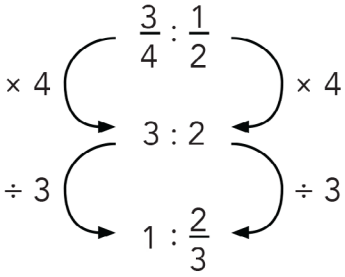
Equivalent Ratios (time : portion of the race)

Manic:



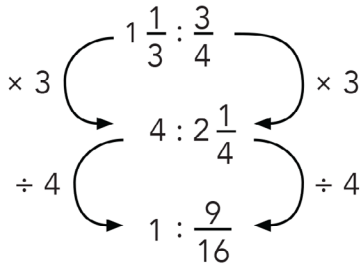
This is less than half of the race.

Crater:



$\frac{2}{3}$ is more than $\frac{1}{2}$ of the race, so Crater is faster than Manic.

Comet:



$\frac{9}{16}$ is less than $\frac{2}{3}$ of the race, so Crater is faster than Comet.

Instructional

Ratio Table

Students may look for and use relationships in their table to compare the dragons' rates. In each table, the highlighted information is provided in the task. Every cell does not need to be filled in to compare the rates of the dragons.

Time (minutes)	**Distance into the Race**		
	Manic	**Crater**	**Comet**
20	$\frac{1}{8}$		
40	$\frac{2}{8}$		
45		$\frac{4}{8}$	
80	$\frac{4}{8}$		$\frac{3}{4}$

Crater completes $\frac{1}{2}$ of the race in less time than Manic's so Crater is faster Manic.
Comet flies farther than Manic in 80 minutes so Comet is faster than Manic.

OR

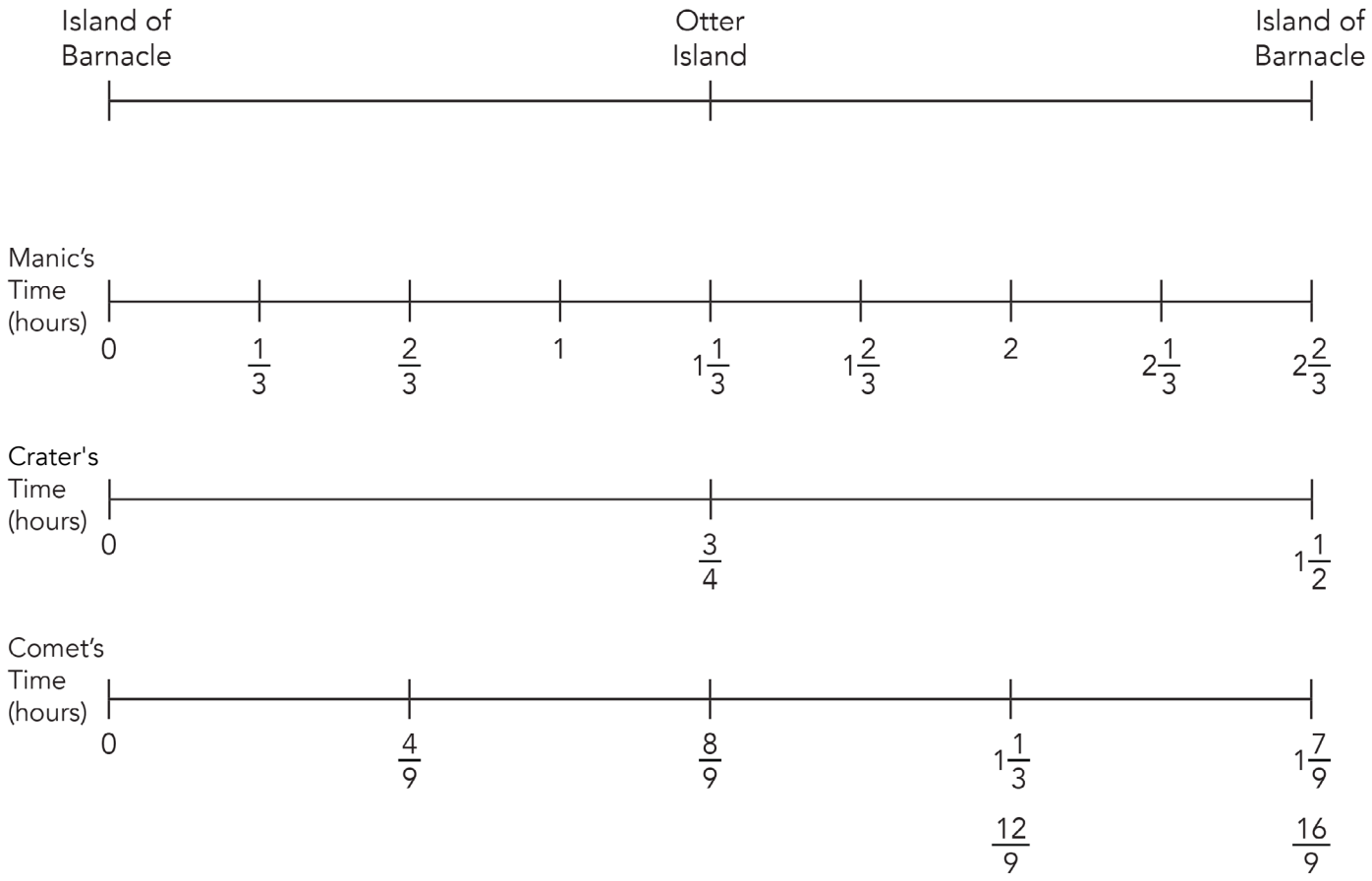
Distance to Otter Island	**Manic's Time** (hours)	**Crater's Time** (hours)	**Comet's Time** (hours)
$\frac{1}{4}$	$\frac{1}{3}$		
$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{8}$	
1	$\frac{4}{3} = 1\frac{1}{3}$	$\frac{3}{4}$	
$1\frac{1}{2}$	2	$\frac{3}{8} + \frac{3}{4}$ or $\frac{3}{4} \cdot 1\frac{1}{2} = 1\frac{1}{8}$	$1\frac{1}{3}$

Based on the last row in the table, it will take Crater the least amount of time to go the same distance as Manic and Comet.

Instructional

Double Number Line

Compare each dragon's time to the portion of the race they've completed. Any of the common points after the start of the race can be used to compare the rates of the dragons.



The number lines can also be used to provide a visual model to help find the time it takes each dragon to complete the whole race:

Manic: $\frac{1}{3} \times 8 = \frac{8}{3} = 2\frac{2}{3}$ hours

Crater: $\frac{3}{4} \times 2 = \frac{6}{4} = 1\frac{1}{2}$ hours

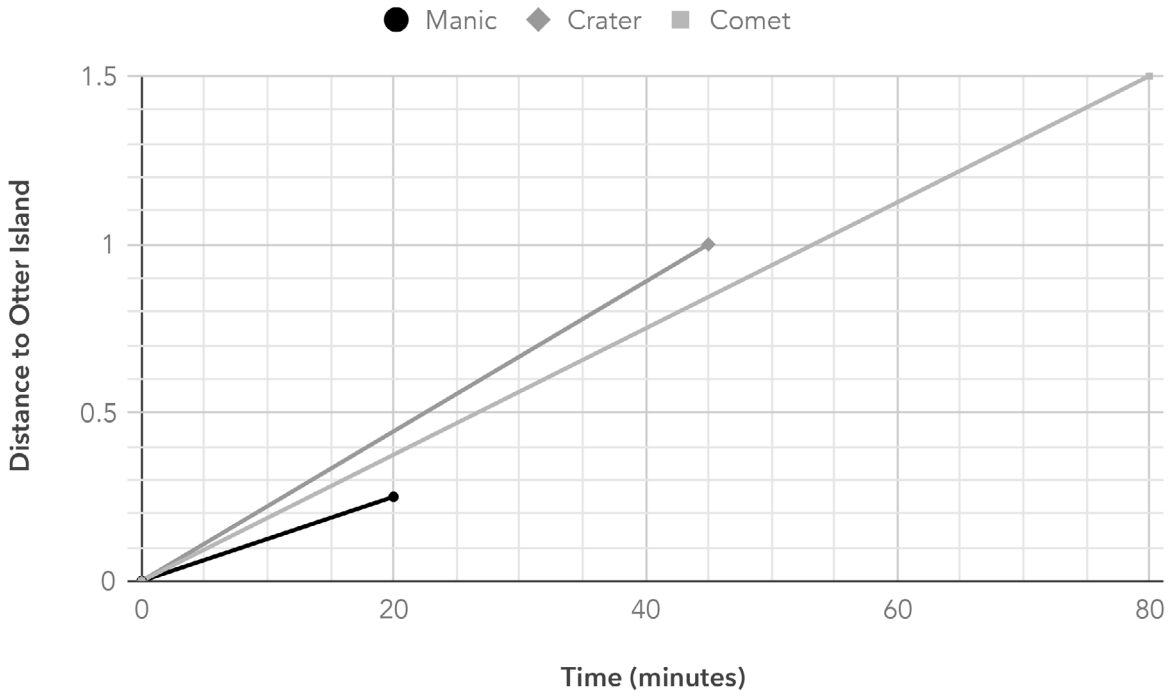
Comet: $1\frac{1}{3} + (\frac{1}{3} \times 1\frac{1}{3}) = 1\frac{7}{9}$ hours

$2\frac{2}{3} > 1\frac{1}{2} < 1\frac{7}{9}$ Crater needs the least amount of time to finish the race and will return to the finish line first.

Instructional

Graph

The beginning of the race can be plotted at (0, 0) for each dragon and connected with a line to the point in the race they were seen.



The graph shows that Crater is moving at a faster rate and will return to the finish line first.

Possible Connections

- A common quantity for either the time or distance the dragons travel allows you to compare their rates.
- When a dragon flies the full distance to Otter Island, this is equivalent to half of the race.
- The dragon that reaches Otter Island first is expected to win the race assuming they continue at their current speeds.
- Crater is in the lead from the beginning of the race, and will hold the lead the entire race.
- The three dragons will never be at the same location other than at the start and end of the race.
- Solve more than one way to verify the answer.
- Relate to a similar task and state a math link.

Engagement Image to Launch Task

Teachers use this resource to pique student curiosity.

Summative

Going Airborne

Peter Pan, Aladdin and the Wicked Witch are debating who can fly faster. To settle the dispute they have decided to have a race around the world. The Jolly Organization for Kartoon Exercise has agreed to monitor the event for fairness and sportsmanship.

After $\frac{1}{2}$ an hour, Peter Pan had traveled $\frac{2}{7}$ of the distance around the world.

After $\frac{2}{5}$ of an hour, Aladdin had traveled $\frac{1}{4}$ of the way around the world.

After $\frac{1}{3}$ of an hour, the Wicked Witch had traveled $\frac{2}{9}$ of the way around the world.

If these athletic cartoons maintain their current speed, who do you predict will make it around the world first? Be sure to provide a clear mathematically supported argument to the Jolly Organization for Kartoon Exercise.

Common Core Task Alignments

Mathematical Practices: MP.1 MP.2 MP.4 MP.5 MP.6

Grade 7 Content Standards: 7.RP.A.1

Planning Sheet

Going Airborne

Common Core Standards and Evidence

7.RP.A.1

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour with 2 being the unit rate.

Exemplars Task-Specific Evidence

This task requires students to find and compare three different rates involving complex fractions to determine which cartoon character will win in a race around the world.

Underlying Mathematical Concepts

- Equivalent ratios
- Comparing fractions
- Benchmark fractions
- Complex fractions

Possible Problem-Solving Strategies

- Unit rate
- Tape diagram
- Double number line
- Ratio table
- Graph

Possible Mathematical Vocabulary/Symbolic Representation

- Equivalent ratio
- Unit rate
- Rate
- Tape diagram
- Double number line
- Ratio table
- Graph
- Coordinates
- x-axis, y-axis
- Independent/Dependent variable
- Formula
- Constant speed
- Divisor
- Quotient
- Greater than
- Less than
- Complex fraction

Summative

Possible Solutions

If the cartoons maintain their current speed, the Wicked Witch is expected to make it around the world first and win the race.

Two different unit rates can be used to solve this problem: the distance around the world per hour or time per trip around the world.

Unit Rate - Distance around the world per hour

Peter Pan: $\frac{4}{7}$ of the Distance Around the World Per Hour
($\frac{2}{7}$ around the world ÷ $\frac{1}{2}$ hour = $\frac{2}{7} \times 2 = \frac{4}{7}$)

Aladdin: $\frac{5}{8}$ of the Distance Around the World Per Hour
($\frac{1}{4}$ around the world ÷ $\frac{2}{5}$ hour = $\frac{1}{4} \times \frac{5}{2} = \frac{5}{8}$)

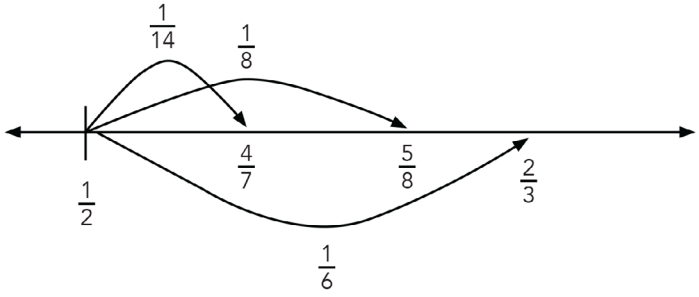
Wicked Witch:
$\frac{6}{9}$ of the Distance Around the World Per Hour
($\frac{2}{9}$ around the world ÷ $\frac{1}{3}$ hour = $\frac{2}{9} \times 3 = \frac{6}{9} = \frac{2}{3}$)

The cartoon's rates can all be compared to the benchmark of $\frac{1}{2}$
Peter Pan: $\frac{1}{2} = \frac{3.5}{7}$ so $\frac{4}{7}$ is half of $\frac{1}{7}$ (or $\frac{1}{14}$) more than $\frac{1}{2}$.

Aladdin: $\frac{1}{2} = \frac{4}{8}$ so $\frac{5}{8}$ is $\frac{1}{8}$ more than $\frac{1}{2}$.

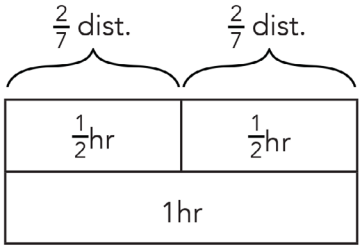
Wicked Witch: $\frac{1}{2} = \frac{1.5}{3}$ so $\frac{2}{3}$ is half of $\frac{1}{3}$ (or $\frac{1}{6}$) more than $\frac{1}{2}$.

$\frac{1}{14} < \frac{1}{8} < \frac{1}{6}$ so $\frac{4}{7} < \frac{5}{8} < \frac{6}{9}$
An open number line can be used to show this relationship:



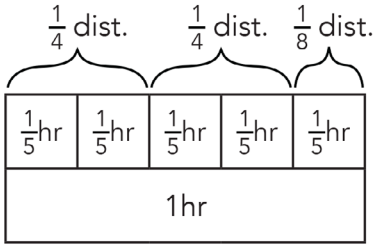
Summative

Peter Pan:



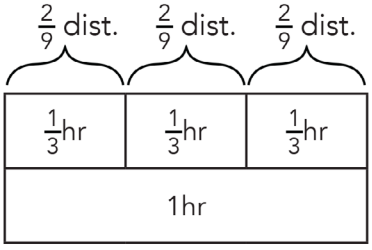
$\frac{2}{7} \times 2 = \frac{4}{7}$ of the distance around the world per hour

Aladdin:



$\frac{1}{4} \times 2 + \frac{1}{8} = \frac{5}{8}$ of the distance around the world per hour

Wicked Witch:



$\frac{2}{9} \times 3 = \frac{6}{9}$ of the distance around the world per hour

Summative

Unit Rate - Time to travel around the world once

It will take Peter Pan $1\frac{3}{4}$ hours to travel around the world
($\frac{1}{2}$ hour ÷ $\frac{2}{7}$ around the world = $\frac{1}{2} \times \frac{7}{2} = \frac{7}{4} = 1\frac{3}{4}$)

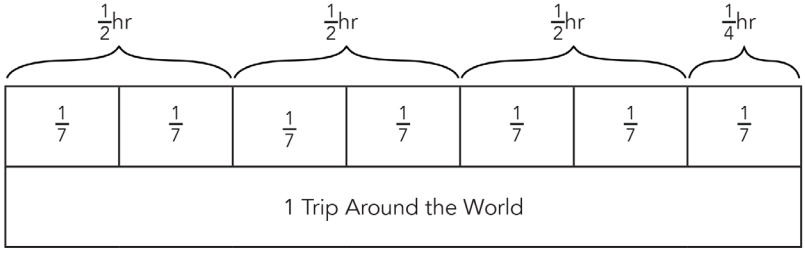
It will take Aladdin $1\frac{3}{5}$ hours to travel around the world
($\frac{2}{5}$ hour ÷ $\frac{1}{4}$ around the world = $\frac{2}{5} \times 4 = \frac{8}{5} = 1\frac{3}{5}$)

It will take the Wicked Witch $1\frac{1}{3}$ hours to travel around the world
($\frac{1}{3}$ hour ÷ $\frac{2}{9}$ around the world = $\frac{1}{3} \times \frac{9}{2} = \frac{9}{6} = 1\frac{3}{6}$)

Since $1\frac{3}{6} < 1\frac{3}{5} < 1\frac{3}{4}$ it will take the Wicked Witch the least amount of time to finish the race.

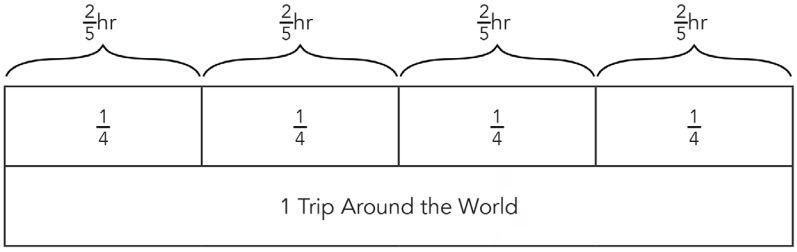
Tape Diagram - Can be used to find and represent each unit rate

Peter Pan:



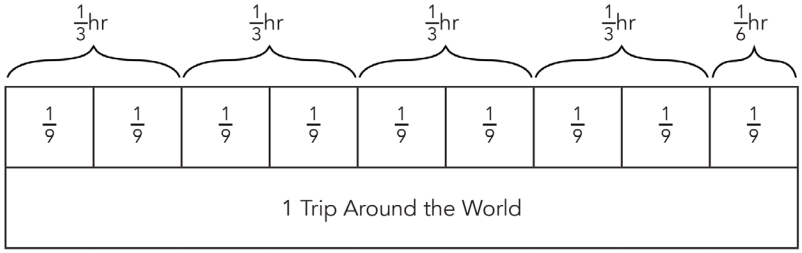
$\frac{1}{2}$ hour × 3 + $\frac{1}{4}$ hour = 1 $\frac{3}{4}$ hours

Aladdin:



$\frac{2}{5}$ hour × 4 + $\frac{8}{5}$ hour = 1 $\frac{3}{5}$ hours

Wicked Witch:



$\frac{1}{3}$ hour × 4 + $\frac{1}{6}$ hour = 1 $\frac{3}{6}$ hours

Summative

Ratio Table

Peter Pan

Time (Hours)	Distance around the world
$\frac{1}{2}$	$\frac{2}{7}$
1	$\frac{4}{7}$
$1\frac{1}{2}$	$\frac{6}{7}$
2	$1\frac{1}{7}$
$2\frac{1}{2}$	$1\frac{3}{7}$
3	$1\frac{5}{7}$

Aladdin

Time (Hours)	Distance around the world
$\frac{2}{5}$	$\frac{1}{4}$
$\frac{4}{5}$	$\frac{1}{4}$
$1\frac{1}{5}$	$\frac{3}{4}$
$1\frac{3}{5}$	1
2	$1\frac{1}{4}$
$2\frac{2}{5}$	$1\frac{1}{2}$

Wicked Witch

Time (Hours)	Distance around the world
$\frac{1}{3}$	$\frac{2}{9}$
$\frac{2}{3}$	$\frac{4}{9}$
1	$\frac{6}{9}$
$1\frac{1}{3}$	$\frac{8}{9}$
$1\frac{2}{3}$	$1\frac{1}{9}$
2	$1\frac{3}{9}$

After 2 hours, Peter Pan could travel $1\frac{1}{7}$ times around the world; Aladdin could travel $1\frac{1}{4}$ times around the world; the Wicked Witch could travel $1\frac{3}{9}$ times around the world.

$1\frac{1}{7} < 1\frac{1}{4} < 1\frac{3}{9}$ so the Wicked Witch is the traveling the fastest. A variety of strategies can be used to compare these rates.

Aladdin is traveling faster than Peter Pan because $1\frac{1}{4} > 1\frac{1}{7}$.

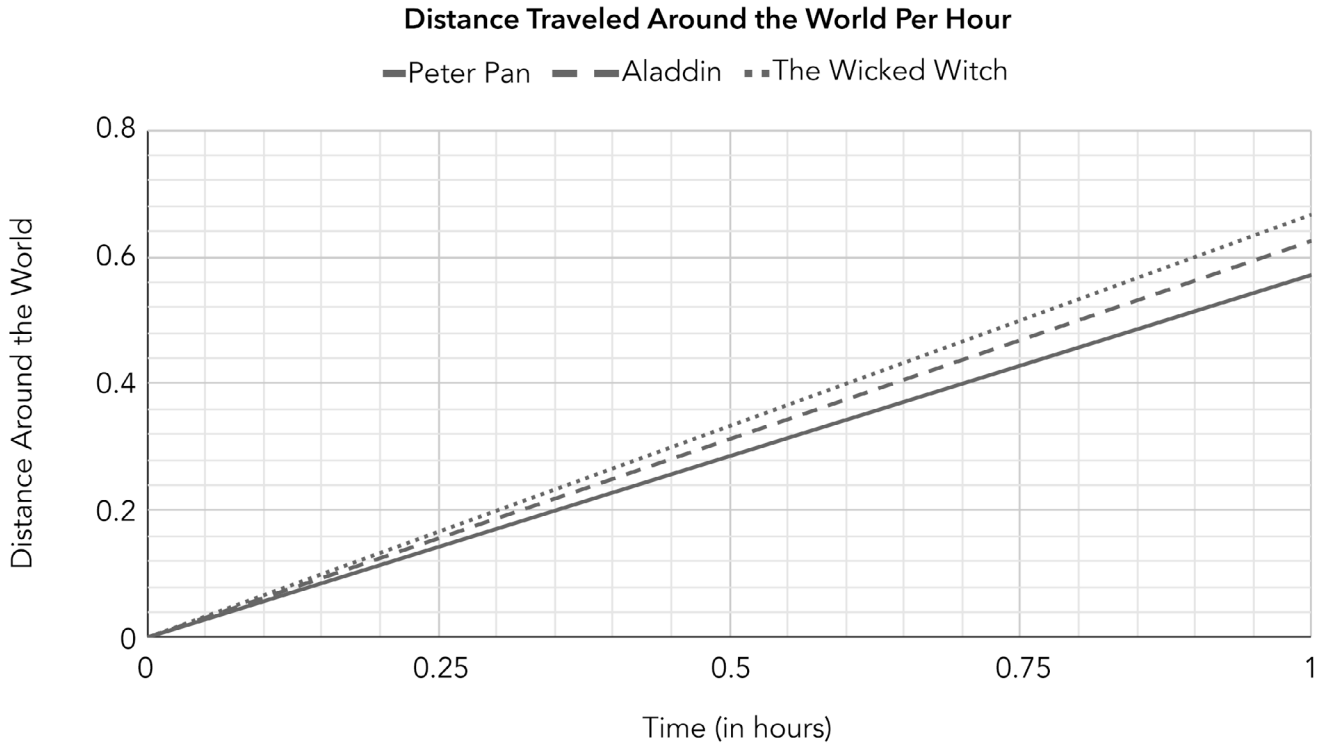
The Wicked Witch is traveling faster than Aladdin because $1\frac{3}{9} > 1\frac{1}{4}$.

Summative

Graph

The information given for each character can be graphed as points on a graph as (time, distance) along with the point (0, 0) for each which represents the start of the race.

The two points for each character can be connected with a line. Each character's speed is represented by the steepness of their line. Since the Wicked Witch's line is the steepest, she is traveling at the fastest rate.



Scaling

Each characters' time can be scaled up to 1 hour to find how far each character has traveled per hour.

Peter Pan: $\frac{1}{2}$ hour × 2 = 1 hour. $\frac{2}{7}$ of the distance × 2 = $\frac{4}{7}$.

Aladdin: $\frac{2}{5}$ hour × 2.5 = 1 hour. $\frac{1}{4}$ of the distance × 2.5 = $\frac{2.5}{4}$ or $\frac{5}{8}$.

Wicked Witch: $\frac{1}{3}$ hour × 3 = 1 hour. $\frac{2}{9}$ of the distance × 3 = $\frac{6}{9}$.

Possible Connections

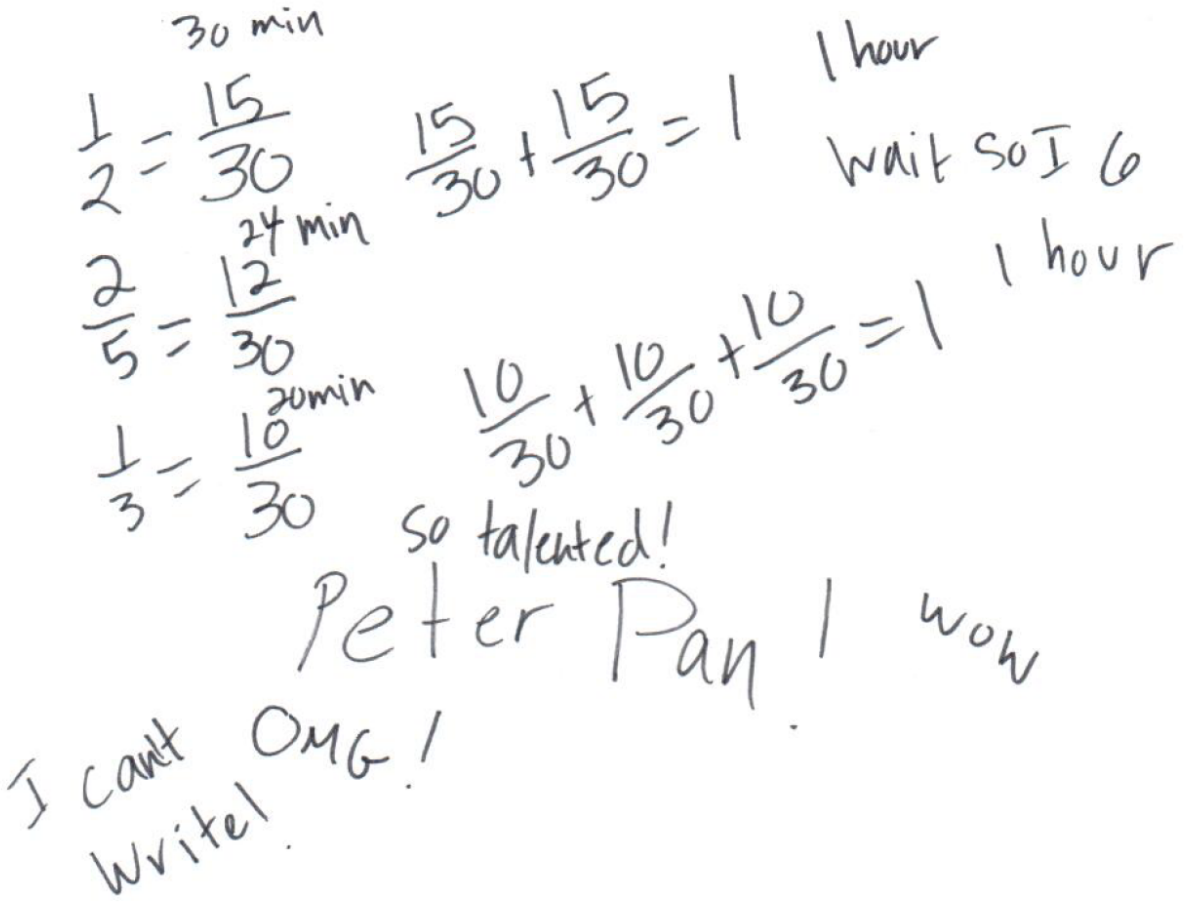
- Speed is found with distance ÷ time.
- Determine how far around the world the remaining characters will be when each one finishes the race.
- The distance around the world is 24,901 miles.
- Determine how much of a head start each character would need for them to tie the race.
- If they only race around the world once, the characters will never be at the same point other than the start.
- Relate to a similar task and state a math link.
- Solve the task another way to verify the answer.

Novice Scoring Rationale

Criteria and Performance Level	Rationales
Problem Solving *Novice*	The student's strategy of changing the hour values to minutes is incomplete and does not lead to a solution. The student states "Peter Pan" which we must assume the student believes is the fastest, which is incorrect.
Reasoning & Proof *Novice*	The student does not demonstrate an understanding of the concept of unit rates associated with ratios of fractions. It appears the student attempts to convert fractional parts of hours to minutes. The student does not consider the distances traveled in this solution process. The mathematical reasoning present does not support their conclusion.
Communication *Novice*	The student does not communicate any purpose for this problem. Limited math language is provided by the student, (min., hour) but significant interpretation is required to determine where these labels connect. The student provides no explanation of his/her approach and only states "Peter Pan" when giving the solution.
Connections *Novice*	The student does not make a mathematically relevant observation.
Representation *Novice*	No attempt is made to make a mathematical representation.

Novice

P/S	R/P	Com	Con	Rep	A/Level
N	N	N	N	N	N



Summative

Apprentice Student 1 Scoring Rationale

Criteria and Performance Level	Rationales
Problem Solving Practitioner	The student's initial strategy of finding common denominators to compare the distance traveled for each character (252) could work to solve the task. The student correctly determines that Peter Pan will take 1 hour and 45 minutes to travel around the world, Aladdin will take 1 hour 36 minutes, and the Wicked Witch will take 1 hour 30 minutes and concludes that the Wicked Witch will travel around the world's fastest. However, the strategy used to reach this solution is not evident.
Reasoning & Proof *Apprentice*	The student demonstrates some understanding of the concept of unit rates associated with ratios of fractions. The student correctly determines the distance traveled around the world by each character's first interval using common denominators. The final solution is correct. The student however provides no mathematical justification for his/her conclusions for how much time it would take each character to complete the race.
Communication *Apprentice*	The student provides a clear purpose for the problem in his/her opening sentence. The student attempts to explain his/her initial approach of finding common denominators, but the mathematical argument lacks detail and significant interpretation of the required approach. The student does not provide his/her approach to how the time traveled was converted into minutes or how the exact time to travel completely around the world was calculated. The student correctly uses formal math language, including fastest, distance, common denominators, minutes, hour, and constant pace.
Connections Practitioner	The student identifies an important situational context within the task when he/she states that "this is all assuming that they each of them keep a constant pace."
Representation Practitioner	The student constructs a table which accurately portrays his/her solutions to the time and distances traveled by each of the contenders in the problem.

Apprentice Student 1

P/S	R/P	Com	Con	Rep	A/Level
P	A	A	P	P	A

Going Airborne

For this Problem I had to find out which of Peter Pan, Aladdin, or the wicked witch was faster so to test that they decided to have a race around the world to find out who is the fastest. To solve this problem I had to find a common denominator and use that to find out who is fastest by traveling around the world. Peter Pan traveled 72/252 around the world in half an hour and finished in 1 hour and 45 minutes. Aladdin traveled 63/252 around the world in 24 minutes and finished in 1 hour and 36 minutes. The Wicked Witch traveled 56/252 in 20 minutes and finished in 1 hour and 30 minutes. This is all assuming that they each of them keep a constant pace.

In conclusion I have figured out that the Wicked Witch is the fastest because she came in first place with a time of 1 hour and 30 minutes, Aladdin came in second, and Peter Pan came in last with the slowest time of 1 hour and 45 minutes. In the table below I showed that stats of each of the contenders.

Contenders	Time	Distance
Peter Pan	1 Hour 45 Minutes	Peter Pan traveled 72/252 of the world every 30 minutes
Aladdin	1 Hour 36 Minutes	Aladdin traveled 63/252 of the world every 24 minutes
Wicked Witch	1 Hour 30 Minutes	The Wicked Witch traveled 56/252 of the world every 20 minutes

Apprentice Student 2 Scoring Rationale

Criteria and Performance Level	Rationales
Problem Solving *Apprentice*	The student's strategy of scaling up the fractional amount of distances traveled in a given time to reach a sum of 1 trip around the world to determine how long it takes each character to travel around the world and which character is traveling the fastest could work to solve the task. The student incorrectly organizes the data for Aladdin, switching the values for time and distance, resulting in an incorrect conclusion that Aladdin will complete the trip in the smallest amount of time.
Reasoning & Proof *Apprentice*	The student demonstrates some correct understanding of unit rates associated with ratios of fractions. The student uses incorrect reasoning when they defined Aladdin's ratio of Distance traveled : Time spent traveling, which is the opposite ratio order used for Peter Pan and the Wicked Witch. The student uses repeated addition to scale up the fractional distances traveled by each character to equal one full trip around the world. The values determined for Peter Pan and the Wicked Witch are correct, while Aladdin's is incorrect.
Communication *Apprentice*	The student attempts an organized, sequenced, and labeled response to communicate his/her work. The student explains the steps to the solution process and explicitly states the solution even though it is incorrect. The use of formal math language is limited, yet appropriate. The student uses incorrect symbolic notation when showing the relationship between distance traveled and the time it takes for each of the characters. The student shows this relationship as $\frac{2}{7}=\frac{1}{2}$ instead of $\frac{2}{7}:\frac{1}{2}$. The student inconsistently includes labels for units of measure (distance, hours, minutes).
Connections *Apprentice*	The student explores the use of rates within the task with additional opportunities to utilize rates to make mathematically based decisions concerning speed of airplanes and the cost of travel.
Representation *Apprentice*	The student attempts to organize the data in a grid for the story and his/her calculations. No categories for the table are provided. The relationships between distances traveled and elapsed time are written incorrectly (ex. $\frac{2}{7}=\frac{1}{2}$ instead of $\frac{2}{7}:\frac{1}{2}$). Unit labels (ex. hours, distance) are missing throughout.

Apprentice Student 2

P/S	R/P	Com	Con	Rep	A/Level
A	A	A	A	A	A

Problem Solving:
The first thing that I did was make a grid, one box for Peter Pan, one box for Aladdin and the last box for the wicked witch. I used the same process for each flyer, here is how I did. I am going to use Peter Pan as an example, I started with 2/7=1/2 then I went to 4/7=1 then 6/7=1 ½ then because 2/7=½, 1/7=1/4 and 6/7+1/7=1 and 1 ½+¼=1 ¾ or 1.75 hours. That is how long it took Peter to fly around the world then I did the same for the other two and it took Aladin 37.5 minutes to fly around the world and the Wicked witch flew around the world in 1.5 hours so Aladin is the fastest.

Reasoning and Proof:
This works because I just kept adding the same time and fraction until I got to 1 which meant they flew around the world once then I recorded the time that it took. Then I went back over my work so there was no room for error.

Communication:
Aladdin is the fastest because he flew around the world in 37.5 minutes when the others flew around in over an hour.

Connection:
This could be used to see how fast planes fly and which are the fastest which would save money of fuel and it would just be more convenient knowing the fly would be faster than other planes that people could be flying in instead.

Representation:

Peter Pan: 2/7=½, 4/7=1, 6/7=1 ½, 1/7=¼. 6/7+1/7+1 and 1 ½ + ¼=1 ¾ so it took peter 1 hour and 45 minutes to fly around the world
Aladin: ⅜=¼, ⅝=½, ⅛=⅛, ⅜+⅛=1 and 4/8+⅛=⅝ so it took Aladin 37.5 minutes to fly around the world
Wicked Witch: 2/9=⅓, 4/9=⅔, 6/9=1, 8/9=1 ⅓, 1/9=⅙, 8/9+1/9=1 and 1 ⅓+⅙=1 ½ so it took the Wicked Witch 1 hour and 30 minutes to fly around the world

Summative

Practitioner Student 1 Scoring Rationale

Criteria and Performance Level	Rationales
Problem Solving Practitioner	The student's strategy of scaling up the time traveled to one hour and scaling up the distance traveled per hour by each contender works to solve the problem. The student's answer "The witch will win, because she is going $0.6\bar{}$dph, which is faster than the other contestants" is correct.
Reasoning & Proof *Practitioner*	The student demonstrates correct understanding of unit rates associated with ratios of fractions. The student converts the time traveled for each character to an hour and then multiplies the distance traveled by the same scale factor. The student then converts the fractions of the distance around the earth traveled into decimal form to compare each contender. The student's approach is systematic with mathematical justification for each contestant.
Communication Practitioner	The student's communication is organized, coherent, and sequenced. The student identifies the problem to be solved, develops a clear approach, and clearly states his/her conclusion. No interpretation is required. The student utilizes formal mathematical language throughout, including distance, per, fastest, dph, ratios, and constant. The work is clear and easy to follow.
Connections Practitioner	The student's connection clarifies the mathematical situation within the task.
Representation *Practitioner*	The student's graph is appropriate for the task and helps portray the solution to the problem. The student clearly defines which line represents each of the characters. All necessary labels are provided.

Practitioner Student 1, Page 1

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

Summary
Who will make it around the world the fastest?

Problem Solving

Current Rates

Peter	1/2 hour	2/7 distance
Aladdin	2/5 hour	1/4 distance
Witch	1/3 hour	2/9 distance

dph = distance per hour

- Peter 2/7 distance in 1/2 hour
  2/7 in 1/2 hour
  x2 x2
  4/7 in 1 hour
  4÷7 = .571, so he is going (.571 dph)

- Aladdin 1/4 distance in 2/5 hour
  1/4 in 2/5 hour
  x2.5 x2.5
  5/8 in 1 hour
  5÷8 = .625, so he is going (.625 dph)

- Witch 2/9 distance in 1/3 hour
  2/9 in 1/3 hour
  x3 x3
  6/9 in 1 hour
  6÷9 = $.\overline{6}$, so she is going ($.\overline{6}$ dph)

Answer
The witch will win, because she is going $.\overline{6}$ dph, which is faster than the other contestents.

Connection
This problem uses ratios, because it gives the distance out of a time, which needs to be simplified to a distance out of a constent time.

Summative

Practitioner Student 1, Page 2

Peter Pan's time can be represented like this-

$\frac{2}{7} : \frac{1}{2}$

distance time

But it needs to be correctly written like this-

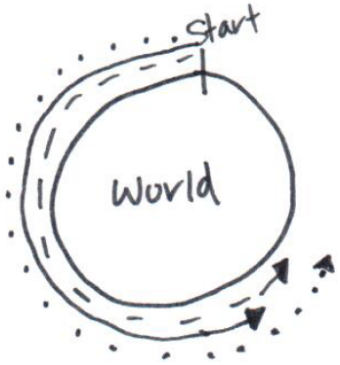
$\frac{4}{7} : 1$

distance time

Which shows the distence out of the time avaliable
The ratios can then be compared to find the fastest

Representation

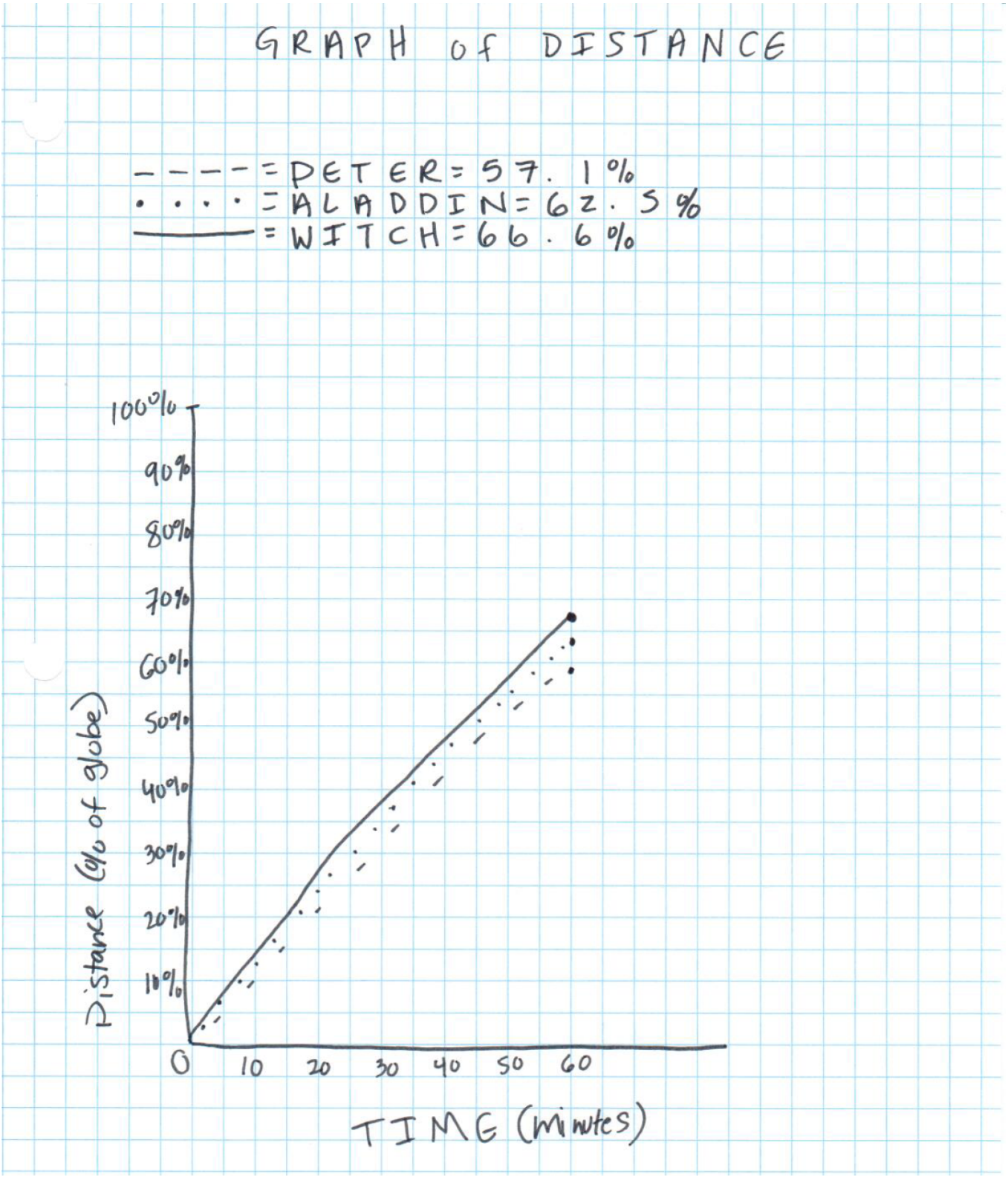
How far each contestent went around the world in 1 hour



— — — = Aladdin 4/7
——— = Peter 5/8
. . . . = Witch 6/9

Summative

Practitioner Student 1, Page 3



Summative

Practitioner Student 2 Scoring Rationale

Criteria and Performance Level	Rationales
Problem Solving Practitioner	The student's strategy of scaling up the distance traveled by the same scale factor required to bring the cartoon's race time to one hour works to solve the problem. The student correctly finds that Peter Pan completes $\frac{4}{7}$ of the total distance around the world in one hour, Aladdin completes $\frac{2.5}{4}$ of the total distance, and the Wicked Witch completes $\frac{6}{9}$ of the distance. The student converts these values to decimals for comparison and correctly determines that the Wicked Witch is traveling the fastest.
Reasoning & Proof *Practitioner*	The student demonstrates correct understanding of unit rates associated with ratios of fractions. The student determines the unit rate of distance traveled per one hour by finding a scale factor which calculates each character's distance traveled after one hour to compare the distance traveled by the characters. The student then converts fractional amounts to decimal equivalents for this comparison. The calculations are accurate and support the solution given.
Communication Practitioner	The student uses an organized, sequenced, and label response to communicate his/her work. The student identifies the problem to be solved, "having a race around the world to see who is fastest," explains the steps to his/her solution process and explicitly states the solution. Formal math language (i.e., scale up, convert, denominator, numerator, compare, decimals, fastest) and symbolic notation are used to consolidate math thinking and to communicate ideas. The work is clear and easy to follow.
Connections Practitioner	The student converts fractional amounts for distances traveled in an hour by each character to decimals for easier comparison.
Representation Practitioner	The student uses several tables, including a ratio table to organize the data and his/her computations in each step of the solution process. The values and labels given are accurate.

Practitioner Student 2, Page 1

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

Peter Pan, Aladdin and the Wicked Witch are having a race around the world to see who is fastest. For simplicity I will refer to the participants as P, A and W respectivly.

Here are the speeds that P, A and W have traveled. So far.

	Time Traveled	Distance around the world
P	1/2 of an hour	2/7 of the world
A	2/5 of an hour	1/4 of the world
W	1/3 of an hour	2/9 of the world

To figure out how far each of them is traveling I will make the time traveld 1 hour and Scale up the distance accordingly.

	Time Traveled (same)			Distance around the world		
P	1/2 x	2	= 1 hour	2/7 x	2	= 4/7 of the world traveled
A	2/5 x	2.5	= 1 hour	1/4 x	2.5	= 2.5/4 traveled
W	1/3 x	3	= 1 hour	2/9 x	3	= 6/9 or 2/3 traveled

Summative

Now that we know the fractions of the time we need to convert them to decimals so we can compare them to each other.

To do this we need to divide the numerator by the denominator

P	$4 \div 7 = 0.57\overline{1}$
A	$2.5 \div 4 = 0.625$
W	$2 \div 3 = 0.66\overline{7}$ - fastest

Since the Wicked Witch is traveling the fastest, we can predict she will finish the race first.

Summative

Expert Student 1 Scoring Rationale

Criteria and Performance Level	Rationales
Problem Solving *Expert*	The student's strategy of scaling up unit fractions to evaluate who will travel around the earth first works to solve this task. The student adjusts this strategy for each of the 3 characters while utilizing this general process. The student correctly determines that it would take Peter Pan 1 hour 45 minutes, Aladdin 1 hour 36 minutes, and the Wicked Witch 1 hour 30 minutes to travel once around the world. The student also utilizes an alternative strategy of double number lines.
Reasoning & Proof *Expert*	The student demonstrates correct understanding of unit rates associated with ratios of fractions. The student provides a rigorous mathematical argument to justify his/her conclusion by using double number lines and calculations. Substantial mathematical evidence is developed by the student as he/she converts the distance traveled for Peter Pan and the Wicked Witch into a unit fraction, allowing a scaling up of the time required to complete a trip around the world. The student then converts fractional parts of an hour to minutes. Computations are accurate and support the solution given.
Communication Expert	The student uses an organized, coherent, and labeled response to communicate his/her approach. The student identifies the problem to be solved, explains the steps to the solution process, explicitly states the solution, and verifies the solution by explaining a second solution strategy. The student utilizes mathematical language and symbolic notation to clearly consolidate and communicate the mathematical ideas for solving this task.
Connections Expert	The student extends his/her solution to a related task, calculating the time it would take if each character took an equal leg of the trip around the world. The student provides significant mathematical justification for his/her conclusion that it would take 1 hour 37 minutes to complete a trip around the world. The student also converts time in fractional parts of an hour into minutes.
Representation *Expert*	The student constructs double number lines to extend his/her thinking to analyze the relationship between distance traveled and the time required to travel that distance. The double number lines verify the solution by showing the complete trip around the world and the time required. The data and labels shown are clear and accurate. The student uses a table to organize data on the connection extension of the problem.

Expert Page 1

P/S	R/P	Com	Con	Rep	A/Level
E	E	E	E	E	E

Who will make it around the world the first ?

Peter pan: if he can get $\frac{2}{7}$ around the world in $\frac{1}{2}$ hour, then it will take him 1 hour and 45 min to get around the world.

Because: if he can travel $\frac{2}{7}$ around the world in $\frac{1}{2}$ of an hour, then he can travel $\frac{1}{7}$ around the world in $\frac{1}{4}$ of an hour (15 min). So now we need to do $\frac{1}{4}$ of an hour (15 min) • 7, because $\frac{7}{7}$ = Whole. 15 min • 7 = 105 min. So it will take peter pan 1 hour & 45 min to get around the world (105 min).

Alladin: if he can get $\frac{1}{4}$ around the world in $\frac{2}{5}$ of an hour, then it will take him 1 hour and 36 min to get around the world

Because: if he can travel $\frac{1}{4}$ around the world in $\frac{2}{5}$ of an hour, we need to multiply $\frac{2}{5}$ by 4 because he is only $\frac{1}{4}$ around the world and he needs to get $\frac{4}{4}$ around the world. so $\frac{2}{5}$ hour • 4 = $\frac{8}{5}$ hour. $\frac{8}{5}$ is equal to 1 hour and 36 min. So it will take Alladin 1 hour and 36 min to get around the world

Wicked witch: if she can get $\frac{2}{9}$ around the world in $\frac{1}{3}$ of an hour, then she can get around the world in $1\frac{1}{2}$ hours.

Because: if she can get $\frac{2}{9}$ around in $\frac{1}{3}$ of an hour, she can get $\frac{1}{9}$ around in $\frac{1}{6}$ of an hour. So we need to do $\frac{1}{6} \times 9$ because we want to get $\frac{9}{9}$ around the world.

Summative

Expert Page 2

Conclusion: The wicked witch can get around the world first because she had the shortest amount of time after she made it around the world.

final times:

Peter pan: 1 hour, 45 min
Aladdin : 1 hour, 36 min
Wicked Witch: 1 hour, 30 min

Connection: if they all worked together, and divided the distance into 3 parts, they would complete the race in 1 hour, 37 min.

Peter Pan's time for his $\frac{1}{3}$

1 hour, 45 min ÷ 3 = 35

35 min

Alladin's time for his $\frac{1}{3}$

1 hour, 36 min ÷ 3 = 32

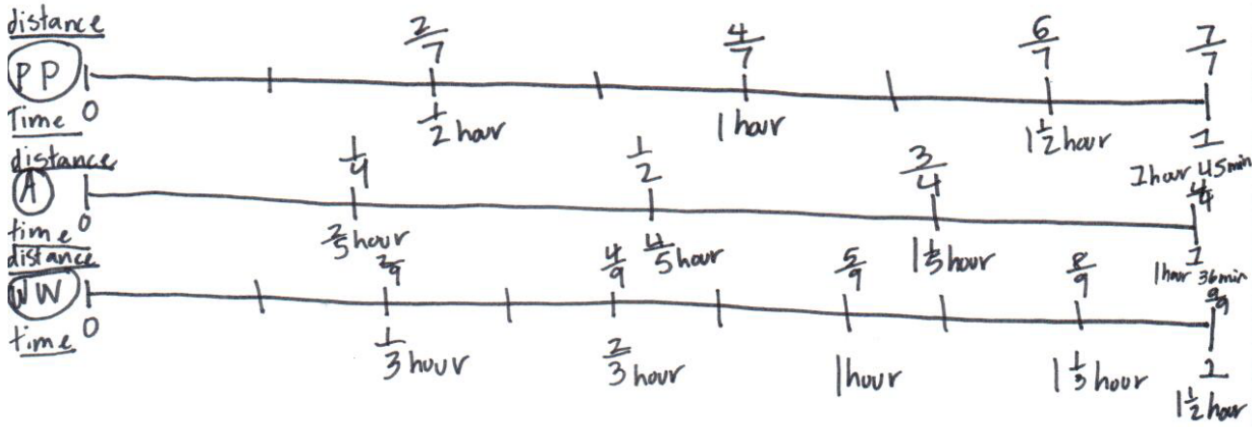
32 min

Wicked witch's time for her $\frac{1}{3}$

1 hour, 30 min ÷ 3 = 30

30 min

35
32
+ 30
97 min (97 min = 1 hour + 37 min)



Summative

Expert Student 2 Scoring Rationale

Criteria and Performance Level	Rationales
Problem Solving *Expert*	The student's strategy of using the relationship between distance, rate, and time (D = R x T) to find the unit rate of distance traveled around the world per hour for each character works to solve the task. The student correctly determines that Peter Pan will travel $\frac{4}{7}$ (.57) of the distance around the world in an hour, Aladdin will travel $\frac{5}{8}$ (.625) of the distance in an hour, and the Wicked Witch will travel $\frac{2}{3}$ ($.66\bar{}$) of the distance in an hour. The student correctly determines that the Wicked Witch will travel around the world the fastest. The student then uses an alternative to find the time it would take each character to travel around the world.
Reasoning & Proof *Expert*	The student demonstrates correct understanding of unit rates associated with ratios of fractions. The student correctly applies the formula of D ÷ T = R to determine the rate (distance traveled per hour) each character is traveling as a fraction of one hour. The student converts fractional amounts to decimals for easier comparison. As an alternative strategy to compare the speed of each character, the student converts the rate per hour to actual time to complete the entire race in hours and minutes. The calculations support the solution given.
Communication Expert	The student uses an organized, sequenced, and label response to communicate work. The student identifies the problem to be solved, explains the steps to the solution process, and explicitly states the solution. The student verifies his/her solution by explaining a second solution strategy. Formal math language and symbolic notation, including rate, formula, per, and equations is used to consolidate math thinking and to communicate ideas. The work is clear and easy to follow.
Connections Expert	The student extends his/her solution of the problem by determining who won the race using two strategies. The student also generalizes the solution process for determining the rate of travel with the algebraic equation $\frac{D}{T}$= R and the time required to travel the entire distance around the world with the equation $\frac{D}{R}$= T.
Representation *Expert*	The student utilizes algebraic equations to provide a symbolic representation to analyze the relationships between distance, time, and rate. The student utilizes two algebraic equations $\frac{D}{T}$ = R and D = T. The student then determines that comparing both rates and times show that the Wicked Witch is the fastest cartoon. Finally, the student constructs double number lines to compare the time that each character needs to complete one trip around the world.

Expert Page 1

P/S	R/P	Com	Con	Rep	A/Level
E	E	E	E	E	E

Problem: 3 people are racing around the world. Each person is a diferent part of the way done the race, and have taken diferent amounts of time to get there. With the info, we want to find who will finish the race first.

Speed: One way to solve this is to figure out who is going faster. To do this we use the formula RT=D (rate · time = distance) and we know time and distance. So we can find the rate (R) by doing $D/T=R$.

Peter Pan: $\overset{T}{1/2}$ hour, $\overset{D}{2/7}$ of the way done: $2/7 \div 1/2 = 4/7$ Rate = 4/7 or .57... of the race per hour

Aladin: 2/5 hour, 1/4 of the way done: $1/4 \div 2/5 = 5/8$ Rate = 5/8 or .625 of the race per hour

Wicked Witch: 1/3 hour, 2/9 of the way done: $2/9 \div 1/3 = 2/3$ Rate = 2/3 or $.\overline{66}$ of the race per hour

---

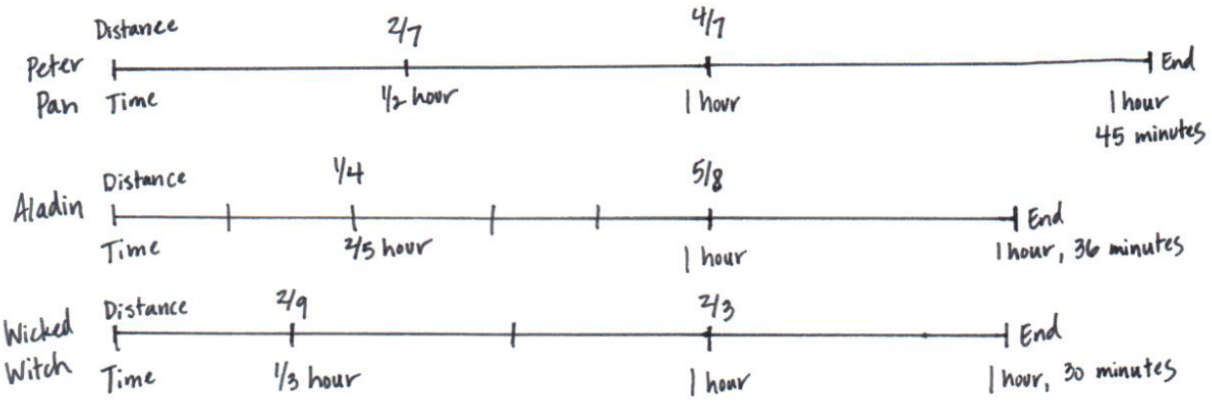
Finish First: The Wicked Witch is going the fastest ($.\overline{66}$ of the race per hour). She will finish first, in second it will be Aladin and in 3rd Peter Pan.

Connection: We can also find how long it will take everyone to finish the race. We use the same formula, RT=D. But now we change around the equation (using algebra) to be $D/R=T$, and we now know the rates and the distance (1).

Peter Pan: $1 \div .57 = 1.75...$ or $\approx$ 1 3/4 of an hour / 1 hour and 45 minutes

Aladin: $1 \div .625 = 1.6$ or 1 3/5 of an hour / 1 hour and 36 minutes

Wicked Witch: $1 \div .\overline{66} = 1.5$ or 1 1/2 hours / 1 hour and 30 minutes



So the Wicked Witch finished 6 minutes before Aladin and 15 minutes before Peter Pan.

Summative