

## Engagement Image to Launch Task

*Teachers use this resource to pique student curiosity.*



## Dog Years

It is said that dogs age seven years for every “people year.” Mason’s dog, Shep, was born on Mason’s eighth birthday. When Mason was nine years old, Shep was seven dog years old. In dog years, how old will Shep be on Mason’s twelfth birthday? Show all your mathematical thinking.

## Dog Years

### Common Core Task Alignments

Mathematical Practices: 1, 3, 4, 6, 7

Grade 4 Content Standards:

4.OA.A.2

### Task

It is said that dogs age seven years for every “people year.” Mason’s dog, Shep, was born on Mason’s eighth birthday. When Mason was nine years old, Shep was seven dog years old. In dog years, how old will Shep be on Mason’s twelfth birthday? Show all your mathematical thinking.

### Alternative Versions of the Task

#### More Accessible Version:

It is said that dogs age seven years for every “people year.” Mason’s dog, Shep, was born on Mason’s eighth birthday. When Mason was nine years old, Shep was seven dog years old. In dog years, how old will Shep be on Mason’s tenth birthday? Show all your mathematical thinking.

#### More Challenging Version:

It is said that dogs age seven years for every “people year.” Mason’s dog, Shep, was born on Mason’s eighth birthday. When Mason was nine years old, Shep was seven dog years old. If Shep is fifty-six dog years old, how old is Mason? Show all your mathematical thinking.

### Common Core Content Standards and Evidence

#### 4.OA Operations and Algebraic Thinking

*Use the four operations with whole numbers to solve problems.*

2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

#### Exemplars Task-Specific Evidence

This task requires students to use multiplication in comparison situations.

#### Underlying Mathematical Concepts

- Multiplicative comparison
- Number sense to 28

### Possible Problem-Solving Strategies

- Model (manipulatives)
- Diagram/Key
- Table
- Number line

### Possible Mathematical Vocabulary/Symbolic Representation

- Model
- Diagram/Key
- Table
- Number line
- Pictograph
- Pattern
- Multiple
- Odd/Even
- Per
- Year, month, day
- Ordinal numbers: 8th, 9th, 10th ...
- Total/Sum
- Dozen

### Possible Solutions

#### Original Version:

In dog years, Shep will be 28 years old on Mason's 12th birthday.

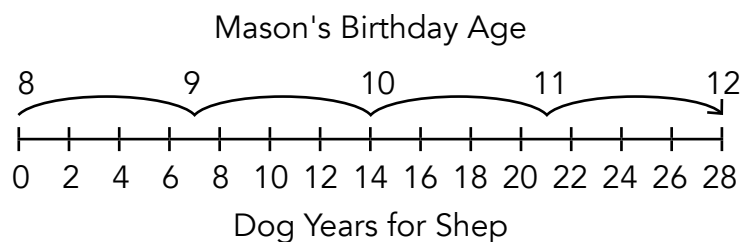
<p><u>Rule</u></p> <p><b>p</b> is people years</p> <p><b>d</b> is dog years</p> <p><math>7 \cdot p = d</math></p>
---

Mason's Birthday	Shep's Age in Dog Years
8th	Just Born
9th	7
10th	14
11th	21
12th	28

$$7 \times 1 = 7$$

$$7 \times 4 = 28$$

$$\begin{array}{r} 12 \\ - 8 \\ \hline 4 \end{array}$$



**More Accessible Version:**

In dog years, Shep will be 14 years old on Mason's 10th birthday.

**More Challenging Version:**

Mason is 16 years old.

**Possible Connections**

Below are some examples of mathematical connections. Your students may discover some that are not on this list.

- Patterns: Mason's age +1, Shep's age in dog years +7.
- Continue to find Mason and Shep's ages after Mason's 12th birthday.
- Rule for Shep:  $7 \cdot \text{"people years"} = \text{dog years}$ ,  $7 \cdot p = d$ .
- If a dog is a dozen people years old, the dog is 84 years old. Dogs don't tend to live as long as people.
- Relate to a similar task and state a math link.
- Solve more than one way to verify the answer.

## Engagement Image to Launch Task

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## Cars on a Ramp

Sabrina and Joel are measuring how far their cars roll down a ramp. Sabrina's car rolls forty-eight inches. Joel's car rolls seventy-two inches. Sabrina says her car rolls four times as far as one foot. Joel says his car rolls two times as far as one yard. Who is correct, Sabrina or Joel? Show all your mathematical thinking.

## Cars on a Ramp

### Common Core Task Alignments

**Mathematical Practices:** 1, 2, 3, 4, 5, 6, 7

**Grade 4 Content Standards:**  
4.OA.A.2

### Task

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### Common Core Content Standards and Evidence

#### 4.OA Operations and Algebraic Thinking

*Use the four operations with whole numbers to solve problems.*

2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

#### Exemplars Task-Specific Evidence

This task requires students to use multiplication in comparison situations. The students also need to know that one foot is 12 inches and one yard is 36 inches.

#### Underlying Mathematical Concepts

- Multiplicative comparison
- Linear measurement:  $12'' = 1'$ ,  $36'' = 1$  yard
- Number sense to 72

#### Possible Problem-Solving Strategies

- Model (manipulatives)
- Diagram/Key
- Chart
- Number line

#### Possible Mathematical Vocabulary/Symbolic Representation

- Model
- Diagram/Key
- Chart
- Number line
- Inch, in., ''
- Foot, ft., '



### Possible Mathematical Vocabulary/Symbolic Representation (cont.)

- Yard, yd.
- Total/Sum
- Amount
- Greater than (>)/Less than (<)
- Most/Least
- Distance
- Per
- Angle
- Degrees

### Possible Solutions

Sabrina and Joel are both correct.

Friend	Car Rolls in Inches	Car Rolls in Feet
Sabrina	48	4
Joel	72	6

6 feet = 2 yards

$$4 \times \square \text{ " } = 48 \text{ "}$$

$$4 \times 1' = 4'$$

$$\square \times 36 \text{ " } = 72 \text{ "}$$

$$2 \times 36 \text{ " } = 72 \text{ "}$$

$$\begin{array}{r}
 12 \\
 4 \overline{) 48} \\
 \underline{- 40} \quad 10 \\
 8 \\
 \underline{- 8} \quad 2 \\
 0 \quad 12
 \end{array}$$

### Possible Connections

Below are some examples of mathematical connections. Your students may discover some that are not on this list.

- The cars roll a total of 120 inches or 10 feet.
- Joel's car travels the most, 2 more feet than Sabrina's car.
- Sabrina's car travels  $1 \frac{1}{3}$  yards or 1 yard and 12 inches.
- Relate to a similar task and state a math link.
- Solve more than one way to verify the answer.
- Using the same ramp makes the distance traveled per car fair as the slope is the same.
- If you raise the ramp, the cars could go faster.
- Angle/ramp/degrees are discussed.

# Novice Scoring Rationales

<b>Criteria and Performance Level</b>	<b>Assessment Rationales</b>
<b>Problem Solving</b> <i>Novice</i>	The student's multiplication would not work to solve this task. The student's answer, "I think they are both correct," does not match the calculations that show 96 for Sabrina and 288 for Joel.
<b>Reasoning Proof</b> <i>Novice</i>	The student does not demonstrate correct reasoning. It appears that the student uses the numbers 48 and 72 from the task and multiplies them by the two and four mentioned in the task. This reasoning would not lead to a correct answer.
<b>Communication</b> <i>Novice</i>	The student does not use any mathematical language or notation.
<b>Connections</b> <i>Novice</i>	The student does not make a mathematically relevant connection. The statement, "I see Joel goes fast," is not linked to any numbers in the problem: One has to assume the student means Joel's car goes fast.
<b>Representation</b> <i>Novice</i>	The student does not use any mathematical representation in her/his solution.

# Novice

P/S	R/P	Com	Con	Rep	A/Level
N	N	N	N	N	N

Is Joel correct or Is Sabrina correct?

Sabrina goes 48. Joel goes 72.

$$48 \times 2 = 96$$

$$40 \times 2 = 80$$

$$8 \times 2 = \underline{+16}$$

$$96$$

$$72 \times 4 = 288$$

$$70 \times 4 = 280$$

$$2 \times 4 = \underline{+8}$$

$$288$$

I think they are both correct.

I see Joel goes fast,

# Apprentice Scoring Rationales

<b>Criteria and Performance Level</b>	<b>Assessment Rationales</b>
<b>Problem Solving</b> <i>Practitioner</i>	The student's strategy of using a chart and division to determine that Sabrina's car does roll four times as far as one foot and Joel's car does roll two times as far as one yard works to solve the task. The student's answer, "Joel is correct," and "Sabrina is correct," is correct.
<b>Reasoning Proof</b> <i>Practitioner</i>	The student demonstrates understanding of the underlying concepts of the task. The student correctly uses division to differentiate multiplicative comparison from additive comparison.
<b>Communication</b> <i>Apprentice</i>	The student correctly uses the mathematical term "in." ( <i>inch</i> ) from the task. The student does not include any other mathematical terms in her/his solution.
<b>Connections</b> <i>Apprentice</i>	The student attempts a mathematical connection by deciding to determine the sum of the inches traveled by the two cars. The student makes a computational error that leads to an incorrect connection of "110 in. – how much they roll together."
<b>Representation</b> <i>Practitioner</i>	The student's use of a chart is appropriate to the task and accurate. Each column is labeled correctly and all entered data is correct.

# Apprentice

P/S	R/P	Com	Con	Rep	A/Level
P	P	A	A	P	A

I have to find out who is correct - Sabrina or Joel. I am going to show the rolls.

Cars	roll
Joels	72 in.
Sabrinias	48 in.

$$36 \overline{) 72} \begin{array}{r} 2 \text{ times} \\ \underline{-72} \\ 0 \end{array}$$

$$12 \overline{) 48} \begin{array}{r} 4 \text{ times} \\ \underline{-48} \\ 0 \end{array}$$

Joel is correct  
Sabrina is correct

$$\begin{array}{r} 72 \text{ in.} \\ + 48 \text{ in.} \\ \hline 110 \text{ in.} \end{array} \text{ - how much they roll together}$$

# Practitioner Scoring Rationales, Student 1

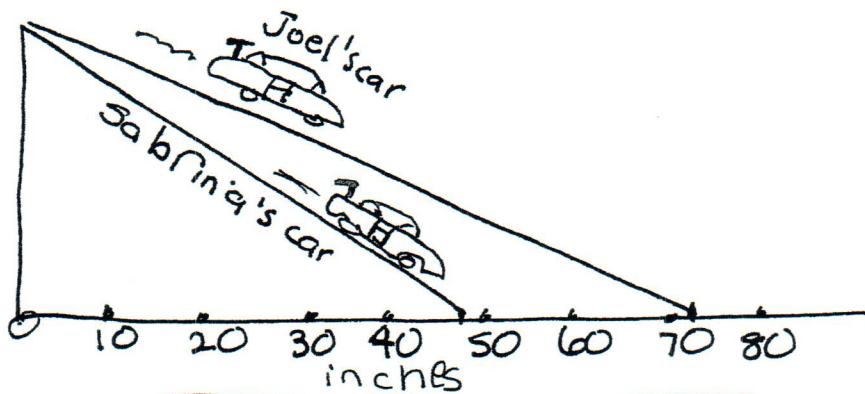
<b>Criteria and Performance Level</b>	<b>Assessment Rationales</b>
<b>Problem Solving</b> <i>Practitioner</i>	The student's strategy of using a diagram, multiplication and division to determine that Sabrina's car does roll four times as far as one foot and Joel's car does roll two times as far as one yard works to solve the task. The student's answer, "Both are right," is correct.
<b>Reasoning Proof</b> <i>Practitioner</i>	The student demonstrates understanding of the underlying concepts of the task. The student correctly uses division and multiplication to differentiate multiplicative comparison from additive comparison.
<b>Communication</b> <i>Practitioner</i>	The student correctly uses the mathematical terms <i>inches</i> , <i>foot</i> , <i>yard</i> from the task. The student also correctly uses terms <i>diagram</i> , <i>more than</i> .
<b>Connections</b> <i>Practitioner</i>	The student makes a mathematical connection by determining that "Joel's car went 24 inches more than Sabrina," and includes the computation " $72 - 48 = 24$ ."
<b>Representation</b> <i>Practitioner</i>	The student's use of a diagram is appropriate to the task and accurate. The cars are correctly labeled and the inch notation is correct.

# Practitioner, Student 1

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

I need to find out who is correct,  
I will make a diagram.

Foot = 12 inches  
Yard = 36 inches



Answer  
Both  
are  
right

Sabrina work $48 \div 4 = 12$ Right	Joe work $36 \times 2 = 72$ Right
--	--

Joel's car went  
24 inches more  
than Sabrina

$$72 - 48 = 24$$

$$\begin{array}{r} 12 \\ 4 \overline{) 48} \\ \underline{-48} \phantom{0} \\ 0 \end{array} \quad \begin{array}{r} 10 \\ 2 \\ \hline 12 \end{array}$$

$$30 + 6 = 36$$

$$2 \overline{) 60} \quad \begin{array}{|c|} \hline 12 \\ \hline \end{array} \quad 72$$

## Practitioner Scoring Rationales, Student 2

<b>Criteria and Performance Level</b>	<b>Assessment Rationales</b>
<b>Problem Solving</b> <i>Practitioner</i>	The student's strategy of using a chart, multiplication and division to determine that Sabrina's car does roll four times as far as one foot and Joel's car does roll two times as far as one yard works to solve the task. The student's answer, "They are both right," is correct.
<b>Reasoning Proof</b> <i>Practitioner</i>	The student demonstrates understanding of the underlying concepts of the task. The student correctly uses division and multiplication to differentiate multiplicative comparison from additive comparison.
<b>Communication</b> <i>Practitioner</i>	The student correctly uses the mathematical terms <i>inches</i> , <i>foot</i> , <i>yard</i> from the task. The student also correctly uses the terms <i>chart</i> , <i>total</i> , <i>dozen</i> .
<b>Connections</b> <i>Practitioner</i>	The student makes the mathematical connections, "48 + 72 = 120 inches the 2 cars went in total," "72 in. is 6 dozen inches," and, "48 in. is 4 dozen inches."
<b>Representation</b> <i>Practitioner</i>	The student's use of a chart is appropriate to the task and accurate. The three columns are correctly labeled and all entered data is accurate.



# Practitioner, Student 2

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

I have to find who is right.  
I will make a chart.

Kid	Inches car went	feet car went
Joel	72 in.	6 ft.
Sabrina	48 in.	4 ft.

key:  
1 foot = 12 inches  
3 feet = 1 Yard  
1 Yard = 36 inches

$$\begin{array}{r} 36 \\ \times 2 \\ \hline 72 \end{array}$$
  

$$\begin{array}{r} 12 \\ \times 4 \\ \hline 48 \end{array}$$

They are both right

48  
+72  
-----  
120 inches  
the 2 cars went in total.

72 in. is 6 dozen inches  
48 in. is 4 dozen inches

## Practitioner Scoring Rationales, Student 3

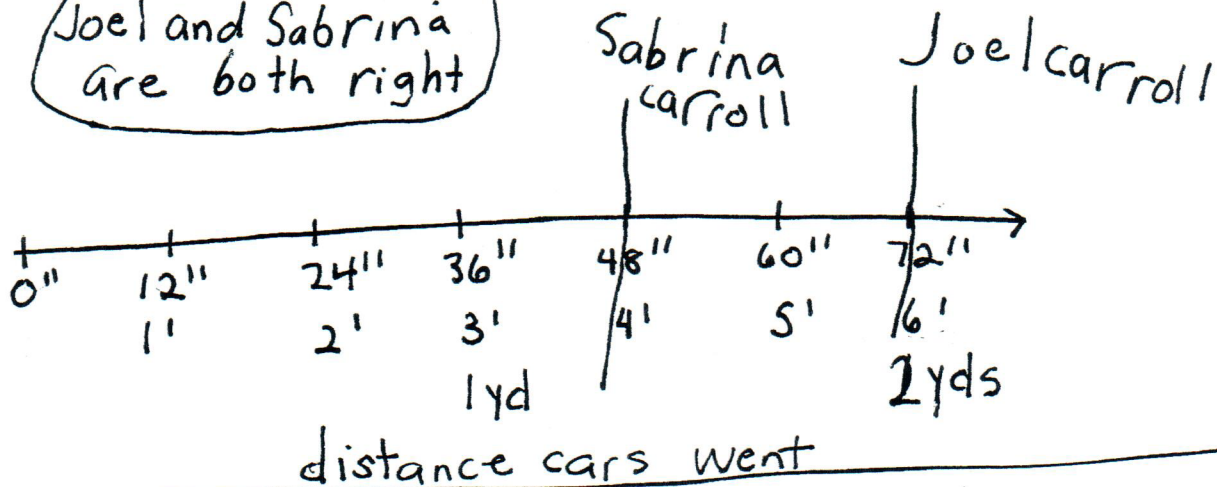
<b>Criteria and Performance Level</b>	<b>Assessment Rationales</b>
<b>Problem Solving</b> <i>Practitioner</i>	The student's strategy of using a number line and multiplication to determine that Sabrina's car does roll four times as far as one foot and Joel's car does roll two times as far as one yard works to solve the task. The student's answer, "Joel and Sabrina are both right," is correct.
<b>Reasoning Proof</b> <i>Practitioner</i>	The student demonstrates understanding of the underlying concepts of the task. The student correctly uses multiplication to differentiate multiplicative comparison from additive comparison.
<b>Communication</b> <i>Practitioner</i>	The student correctly uses the mathematical terms <i>inches</i> , "yds" ( <i>yards</i> ) from the task. The student also correctly uses terms <i>number line</i> , <i>distance</i> , <i>even number</i> . The student correctly uses the mathematical notation 'and'.
<b>Connections</b> <i>Practitioner</i>	The student makes a mathematically relevant observation, "I notice both cars roll an even number of inches."
<b>Representation</b> <i>Practitioner</i>	The student's use of a number line is appropriate to the task and accurate. All necessary labels are provided and the entered data is accurate.

# Practitioner, Student 3

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

I need to find out who is right. I will make a number line.

Answer  
Joel and Sabrina  
are both right



$$12 * 4 = 48 \quad 4 * 10 + 4 * 2 = 40 + 8 = 48 \quad 36 * 2 = 2 * 30 + 2 * 6 = 60 + 12 = 72$$

I noticed both cars roll an even number of inches.

I noticed I made my number line very accurate and put in all the labels.

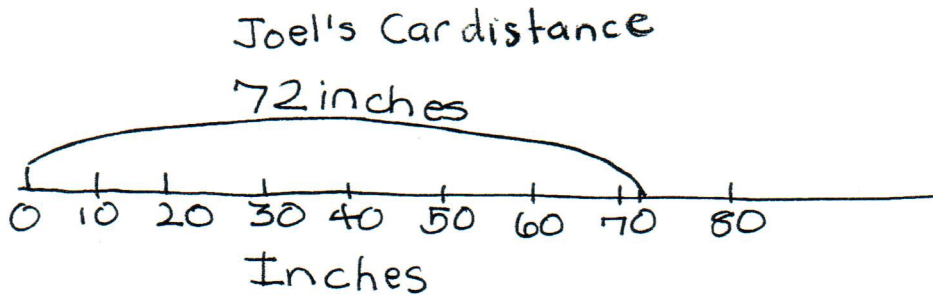
# Expert Scoring Rationales

Criteria and Performance Level	Assessment Rationales
<b>Problem Solving</b> <i>Expert</i>	The student's strategy of using number lines and multiplication to determine that Sabrina's car does roll four times as far as one foot and Joel's car does roll two times as far as one yard works to solve the task. The student's answer, "Both Joel and Sabrina are correct exactley," is correct. The student also uses an alternate strategy of a chart and division. The student also brings prior knowledge of mixed fractions to the task.
<b>Reasoning Proof</b> <i>Expert</i>	The student demonstrates understanding of the underlying concepts of the task. The student correctly uses multiplication to differentiate multiplicative comparison from additive comparison. The student justifies her/his answer by using an alternate strategy of a chart and division to arrive at the same answer.
<b>Communication</b> <i>Expert</i>	The student correctly uses the mathematical terms <i>inches</i> , <i>foot</i> , <i>yard</i> from the task. The student also correctly uses terms <i>number line</i> , <i>distance</i> , <i>key</i> , "tabe," ( <i>table</i> ), <i>division</i> , <i>even</i> . The student correctly uses the mathematical notation " <i>,</i> ", $3 \frac{1}{3}$ , $3 \frac{12}{36}$ .
<b>Connections</b> <i>Expert</i>	The student makes the mathematical connections, "Sabrina was only 24 inches away from Joel's car roll," "or 2 feet away from Joel's car roll," and, "all numbers are even." The student uses arrows to point to the even numbers 48", 4', 72", 6'. The student makes the Expert connection of verifying that her/his answer is correct by using division and a chart. The student states, "I get 4 and 2 again for times so it is correct." The student also determines that ten feet is equivalent to $3 \frac{1}{3}$ yards and 120 inches is equivalent to $3 \frac{1}{3}$ yards.
<b>Representation</b> <i>Expert</i>	The student's use of number lines is appropriate to the task and accurate. Each number line is titled and correctly labeled. 72 and 48 inches are correctly indicated. The student's chart is also appropriate to the task and accurate. All labels are indicated and the entered data is correct. The student uses her/his chart to verify that the data on the number lines and her/his answer is correct.

# Expert

P/S	R/P	Com	Con	Rep	A/Level
E	E	E	E	E	E

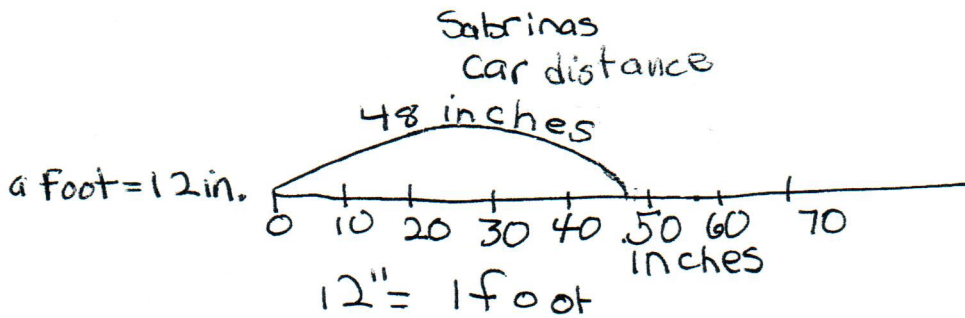
I need to figure who is correct.  
I will make a number line.



Key  
11 inches  
1 foot  
ft  
↓  
I will use both

a yard = 36 in.

Joel says his car rolled 2 times as far as one yard

$$\begin{array}{r} 36'' \\ \times 2 \\ \hline 72'' \end{array}$$


12" = 1 foot

$$\begin{array}{r} 4 \\ \times 12 \\ \hline 48'' \end{array}$$

Sabrina said hers rolled 4 times as far as 1 foot.

Answer - Both Joel and Sabrina are correct - exactly  
connection - Sabrina was only 24 inches away from Joel's car roll  
or 2 feet away from Joel's car roll

$$\begin{array}{r} 6/2 \\ -48 \\ \hline 24 \end{array}$$

## Expert (cont.)

I have to justify I am correct, I will use a table and division this time.

$$12 \text{ in} \overline{) 48 \text{ in}} \begin{array}{r} 4 \rightarrow 4 \text{ times} \\ -48 \\ \hline 0 \end{array}$$

$$36 \text{ in} \overline{) 72 \text{ in}} \begin{array}{r} 2 \rightarrow 2 \text{ times} \\ -72 \\ \hline 0 \end{array} \quad 1 \text{ yard} = 36 \text{ in.}$$

all numbers are even 0

Person's car	Sabrina	Joel
" car rolls	48"	72"
1 car rolls	4'	6'

I get 4 and 2 again for times so it is correct.

$$4' + 6' = 10' \quad 3' \overline{) 10'} = 3 \frac{1}{3} \text{ yds.}$$

$$\begin{array}{r} 3 \\ -9 \\ \hline 1 \end{array}$$

1 yd.

$$48'' + 72'' = 120''$$

$$36'' \overline{) 120''} \begin{array}{r} 3 \\ -108 \\ \hline 12 \end{array}$$

This is  $3 \frac{12}{36} = 3 \frac{1}{3} \text{ yds}$