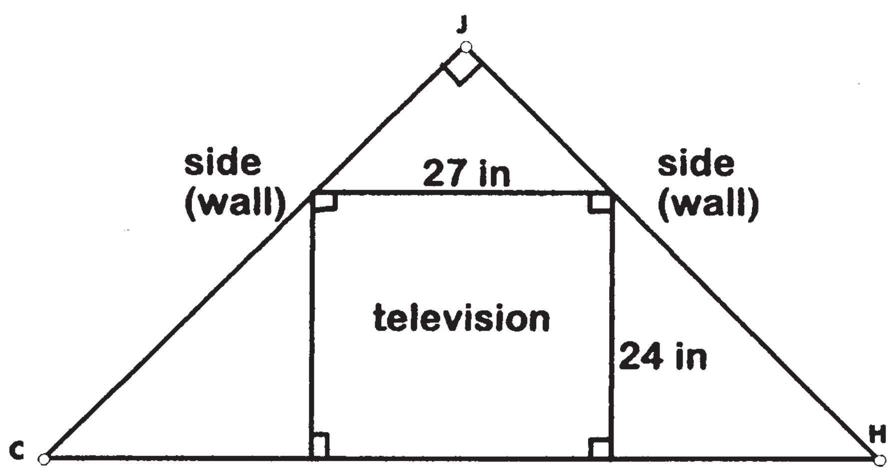
Entertainment Center

In designing a new corner cabinet for our family room, my family and I had to figure out how deep to make it so that the TV we currently have would fit. We want the new cabinet to be the same length on each side (along the two walls).

Here is an overhead view:



How long should each side of the cabinet be? Show all of your calculations and explain how you approached and solved this problem.

Entertainment Center

Common Core Task Alignments

Mathematical Practices: 1, 2, 3, 4, 5, 6, 7, 8

High School Content Standards:
G-SRT.5, G-SRT.8

Suggested Grade Span

Grades 9–10

Task

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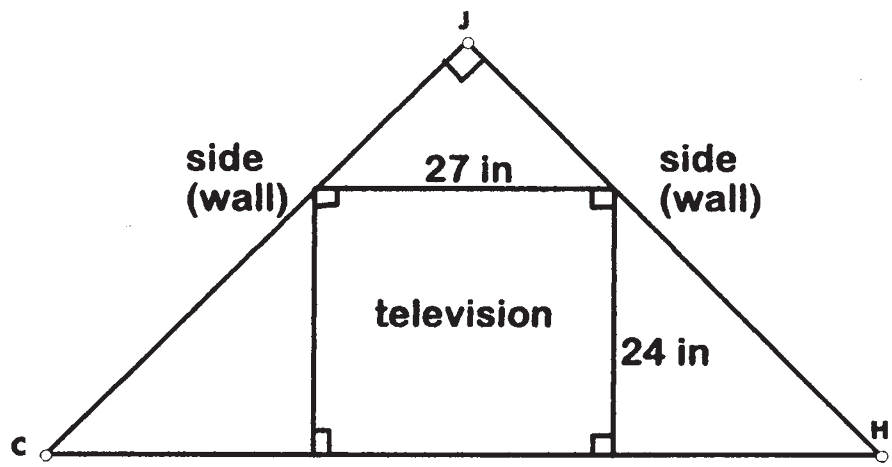


How long should each side of the cabinet be?

Show all of your calculations and explain how you approached and solved this problem.

Context

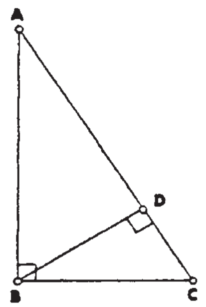
In exploring similar and special right triangles, a variety of approaches were used. The unit began with a discussion of different figures on the overhead that contained different types of similarity and lack thereof, allowing us to define what it is that we mean by indicating that figures are similar.

Once a common understanding was reached, we went to triangles and discovered:

a) a segment parallel to one side of a triangle breaks it into proportional lengths and similar triangles;

b) a segment which connects the midpoints of two sides of a triangle is parallel to the third side and has a measurement equal to one half the length of the third side.

A journal entry was done with the following diagram included:



"Given the drawn triangle relationships, show (give a convincing argument) that the three triangles are similar or not."

We explored the relationships in this diagram using other problems similar to the enclosed task in groups, with the enclosed task being the one used for assessment.

The Pythagorean Theorem was already familiar to the students, so we used it to discover the special right triangle relationships for the 45–45–90 and 30–60–90 triangles.

Before assigning this task, we did have a few days of introduction into trigonometry. This consisted of looking at similar triangles as a class and recognizing that the corresponding ratios of sides always stayed the same. They concluded that these ratios could be related to one of the angles that was the same as another in a similar right triangle. Scientific calculators were introduced, as well as trigonometric tables. Students enjoy working with technology, and it helped us to get through the drudgery of reading trigonometric tables.

What This Tasks Accomplishes

This tasks puts the student in the role of designer, using specifications from a diagram. They must employ (a variety of) techniques and develop appropriate strategies for solving the problem. A meta-cognitive aspect is built into the task by requiring an explanation of the approach and consequent solution.

What the Student Will Do

The students work individually on this problem during a class period. They must show all their work and then verbalize their work and the results. The student is free to choose a variety of approaches and tools.

Time Required for the Task

The students had 30 minutes to complete the task, and it seemed sufficient.

Interdisciplinary Links

As with most of my performance tasks, this one comes from my real-life experience. I believe my students enjoy that connection and work hard on the problems I have had to wrestle with myself.

Teaching Tips

The students enjoyed the task for the most part, though some of my accelerated students had difficulty getting started because they could not see an easy way to an answer. Some students finished early, and I encouraged them to keep writing and to look for alternative approaches. To my surprise, many students asked for scientific calculators to work on this task, even though I had originally intended for it to be a similar and special right triangle problem. Not much time had been spent on trigonometry prior to the task, so it pleased me to find some wanting to give it a try.

Concepts to be Assessed and Skills to be Developed

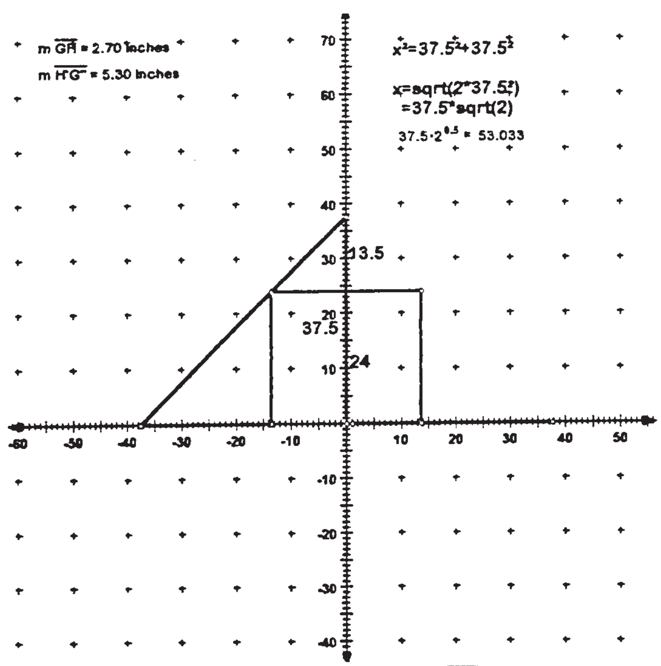
- Pythagorean Theorem
- Ratios in "special" right triangles (45–90–45)
- Generating and solving proportions
- Using right triangle trigonometry
- Using trigonometric functions on the calculator
- Problem Solving
- Communication/Writing Mathematics

Suggested Materials

Calculators, graph paper, trig tables, rulers, protractors, computer with software such as the Geometer's SketchPad.

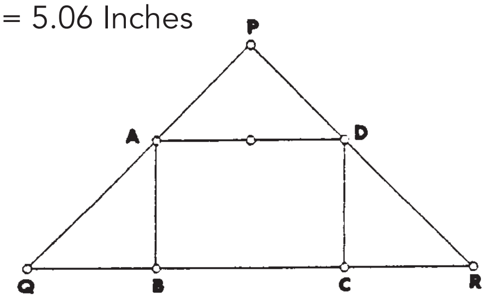
Possible Solutions

1. Using Geometer's SketchPad®, one can set up this problem in a coordinate system. If a proper scale is used the distances can simply be measured. This kind of solution would show that the student understands the symmetry of this problem and makes handy use of this. Even though many students identify the cabinet to be an isosceles triangle, most of them focus on the smaller interior triangles and not on the symmetry in the whole. A coordinate approach might stimulate students to use the symmetry of the problem.



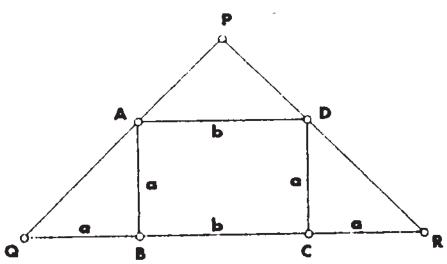
2. RD = 2.06 Inches
   DP = 1.5 Inches
   QP = 3.58 Inches
   QR = 5.06 Inches

QP2.2 = 25.604 inches 2
QR2 = 25.604 inches 2



By drawing any situation like the one proposed in the drawing one can use the Geometer's SketchPad® to discover that the Pythagorean Theorem is useful here, as illustrated on the left. Of course, students can do an accurate drawing by hand and verify the theorem.

This can be generalized in the following situation:



Since PD= $\frac{1}{2}b\sqrt{2}$ and DR= $a\sqrt{2}$ it follows that:

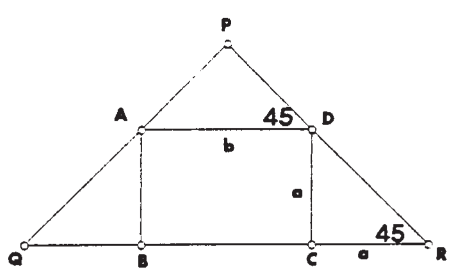
PR= $\frac{1}{2}b\sqrt{2}+a\sqrt{2}=\left(\frac{1}{2}b+a\right)\sqrt{2}$

If you observe these results in triangle PQR it follows that:

$$PR^2 * 2 = QR^2 \Rightarrow PR^2 = \tfrac{1}{2}(b+2a)^2 \Rightarrow$$
$$PR = \sqrt{\tfrac{1}{2}} * (b+2a) \Rightarrow PR = \sqrt{2} * (\tfrac{1}{2}b+a)$$

3. Next follows a result using trigonometric ratios:

sin (45°) = 0.707
cos (45°) = 0.707
$0.5 \cdot (2)^{0.5} = 0.707$



Point of interest here is that triangles APD and DCR have different parts given, the hypotenuse and a leg respectively. Since the value for the sine and cosine of 45 degrees are the same a student could end up with a correct answer using an incorrect choice of function. This makes the written work of the student more valuable.

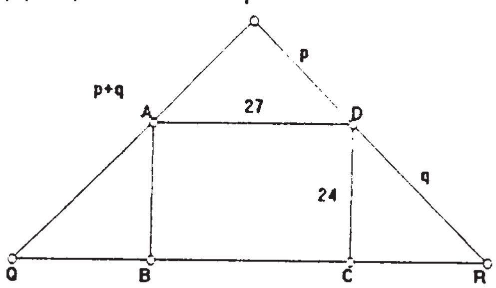
These are possible solutions:

$$\frac{PD}{b} = \cos(45) \Rightarrow PD = b * \tfrac{1}{2}\sqrt{2} \quad (\Delta\ \mathrm{APD})$$

$$\frac{a}{DR} = \cos(45) \Rightarrow DR = a * \sqrt{2} \quad (\Delta \mathrm{RDC})$$

$$\Rightarrow PR = \sqrt{2} * (\tfrac{1}{2}b + a)$$

4. Using proportions in this case will be algebraically cumbersome, but might be an appropriate challenge for some advanced students.



Students quickly find that QR=75. We will develop two proportions to solve for p and q so we can find QP = RP = p + q.

In comparing triangles PAD and PQR we come to the following proportion

$$\frac{27}{75} = \frac{p}{p+q} \qquad (1)$$

In comparing triangles PAD and CDR we come to the following proportion

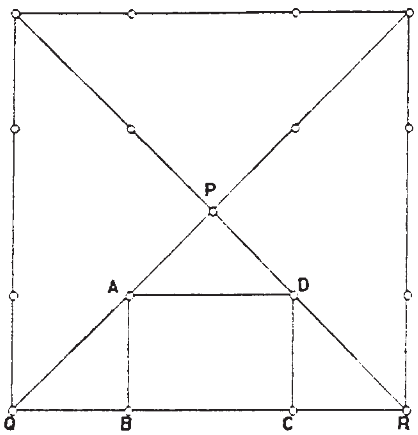
$$\frac{24}{p} = \frac{q}{27} \qquad (2)$$

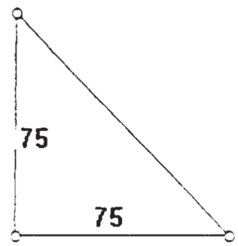
In comparing triangles PQR and CDR we come to the following proportion

$$\frac{24}{p+q} = \frac{q}{75} \qquad (3)$$

Any set of two of these equations will provide the appropriate results. However, equation (2) is particularly suited for substitution purposes.

5. The next solution is very quick and elegant. It requires the student to extend the figure in question beyond itself. If one constructs a square with four of these cabinets then each side of this square would be 75 inches. The sides of the cabinet would then be half the measure of the diagonal, as illustrated in the figure below.





The diagonal will measure $75\sqrt{2}$

Therefore the cabinet side will measure $37.5\sqrt{2}$

Extensions

Some extensions are possible to this problem. You can ask students to take the thickness of the wood into consideration when answering this problem. Also what about beveling the cuts? And how could an entire cabinet be made from 4 feet by 8 feet sheets of plywood with the least amount of waste?

Task-Specific Assessment Notes

Novice: The use of correct formulas will be minimal or non-existent. The written explanations will express confusion and incorrect information. The diagrammed work will be sparse and incorrect. Sometimes, there will be no final answer given.

Apprentice: The Apprentice will have some accurate formulas to find partially correct solutions to the problem. The written explanations usually include a point at which the student becomes stuck. The diagrammed work will include some incorrect information at times.

Practitioner: The Practitioner will use accurate formulas to find correct solutions to the problem. Their written explanations will be clear and straightforward. The diagrammed work will be accurate.

Expert: The Expert will use multiple approaches to the task to find correct solutions. Written explanations will show the student's thoughts clearly. The diagram usage will be solid and correct.

Novice – Sample 1

~~$24^2 + 24^2 = x^2$~~

~~$576 + 576 = 1152$~~

~~$\sqrt{1152} = 33.941$~~

$24 \cdot 3 = 72$

$27 \div 2 = 13.5$

$72 + \cancel{13}^{9}.5 = \cancel{61.5}$

81 in

Use of correct formulas is minimal.

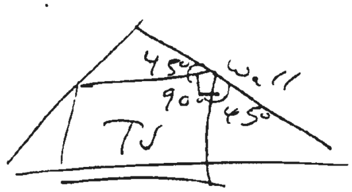
Arbitrary manipulation of givens.

Novice – Sample 1 (cont.)

Since you wanted the length of each side to be the same, ~~at~~ the small triangles with the left + right side of the T.V. have to be ≅. You would also want to ~~have the~~ T.V. ~~centered~~ inside the cabinet, so all of the unknown legs of the small △'s are ≅ with the known legs. (24). also, because of ~~that~~, I knew that the larger triangle was a 45°, 45°, 90° △*

The sides are 1/3 the hypotenuse. So I calculated the legs of the larger △, + the hypotenuse ~~of the~~ smaller △, + I added them up.

* also to prove this, was the fact that the middle 90° of the 180° line was taken up by the TV, so the ~~opposite~~ adjacent angles had to be 45°.



There is confusion about ratios in special right triangles.

Confusion about similarity and congruence is clear from the use of symbols.

Novice – Sample 2

$\cancel{576 + x^2 = x^2}$

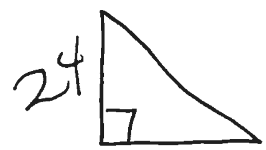
$\frac{24}{x} = \frac{27}{x}$

$24y = 27y$

$y = 1.125y$

$24(1.125y) = 27y$

$27.24y = 27y$



$\cos 90^\circ = \frac{24}{x}$

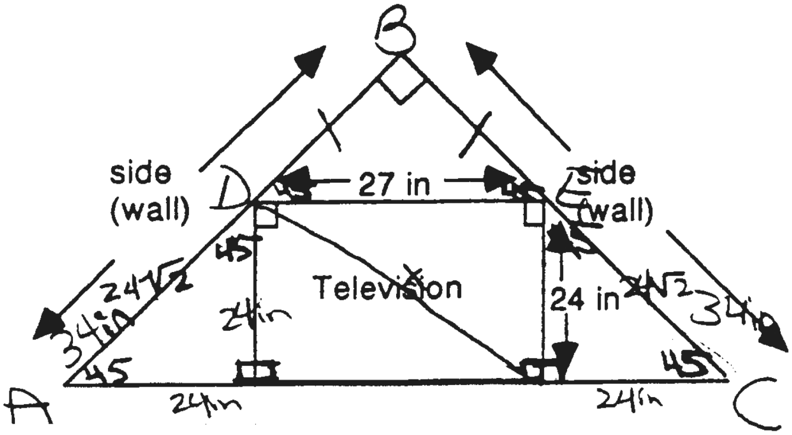
Novice – Sample 2 (cont.)

I couldn't figure this problem out
I hoolced for similar Δ's, but there were none There wasn't enough info for pathagery thereom. And I tried doing proportions, but I could not figure out what to put with what. As for as I could tell, it would be impossible to do ~~this~~ this problem with proportions

The student knows the "buzz–words," but cannot relate any of it to this problem.

Very typical for a Novice: The student has only vague notions about the problem and cannot act on these.

Apprentice – Sample 1



$y, x, \sqrt{2}\,x$

~~Tan 45°~~

$\sin 45° = \frac{24}{x}$

$\boxed{x = 34in}$

$\triangle ABC$ & $\triangle DBE$ are similar

$\cancel{\frac{AD}{AB} = \frac{EC}{CB} \quad AD \times CB = AB \times EC}$

$\frac{AD}{DB} = \frac{EC}{EB} \quad AD \times EB = DB \times EC$

$34 \times x = x \times 34$

$34x = 34x$

There is proper ratio for 90, 45, 45 triangle as well as proper trigonometric ratio.

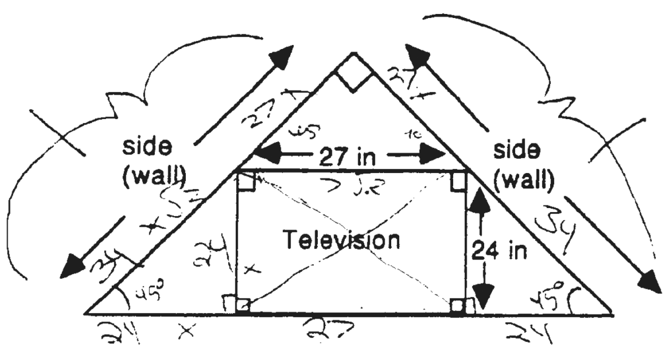
The student cannot relate partial results to the problem to be solved in this task.

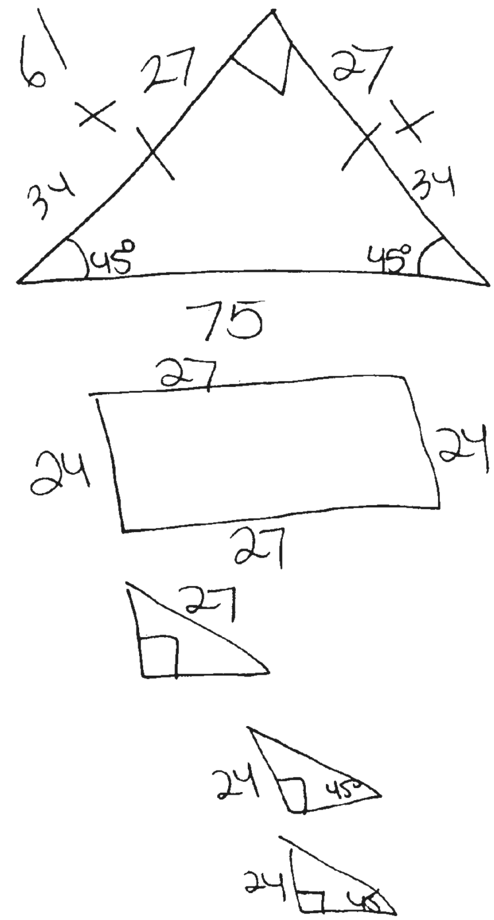
Apprentice – Sample 1 (cont.)

I'M STUCK - I figured out that the $\triangle$s are similar - but after that I got nothing? I tried to do a proportion - but I couldn't figure out one that would work? I want to know if $\overline{DE}$ is $\parallel$ to AB or if ~~the~~ it is a bisector?

The student is stuck and cannot connect partial results. The student asks a question, but does not indicate how an answer to this would be helpful.

Apprentice – Sample 2





45 45 40

x x $\sqrt{2}x$

$24+27+24$

75

$24^2 \times 24^2 = x^2$

576×576

$1152 = x^2$

There is proper use of ratios and Pythagorean Theorem, but it is inconsistent.

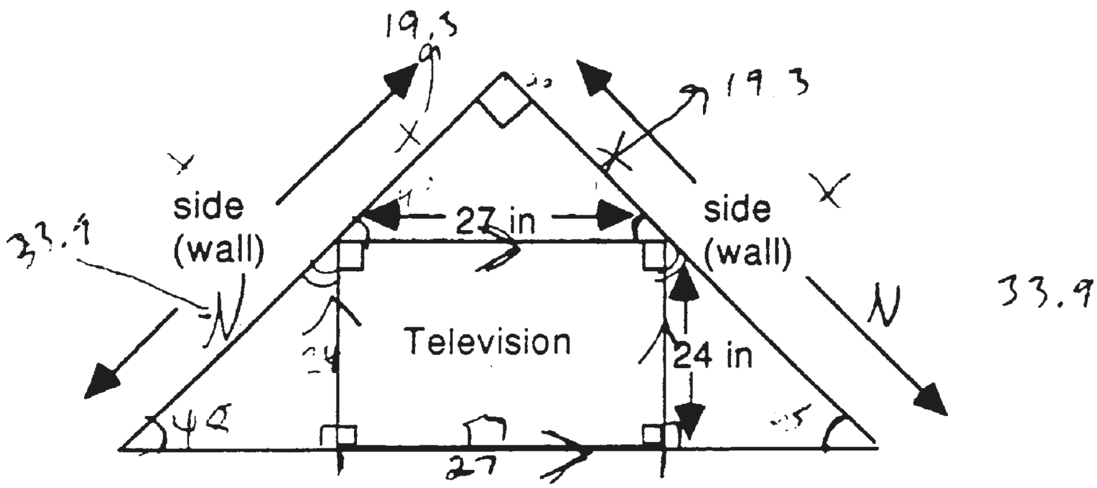
The student does not use partial results to the problem in this task.

Apprentice – Sample 2 (cont.)

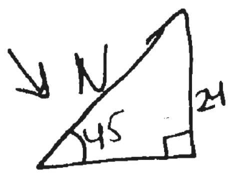
I first tried to find the
front of the cabinet. I knew the T.V.
is 27 in. across. I figured out that the
big triangle was an isosceles The base
angles must be 45°. That 45° angle is
about part of the smaller triangle. The
2 smaller triangle must be isosceles.
I figured that the front of
the cabinet is 75° in. I did the
pathagerom theroum to figure
out a part of the side Part of
the side is 34 + New I
am stuck

The student is capable of figuring out small parts related to the problem, but s/he does not relate these results to each other.

Practitioner – Sample 1



53.2



sin = 5 $\quad \frac{24}{y}$

19.5 ×

$\sqrt{2}\ x\ (27)$

$x = 19.3\text{in}$

$\frac{x(14\times5)}{(.7071)} = \frac{74}{.7071}$

$x = 33.9$

$\frac{27}{\sqrt{2}} = x$

$\frac{27}{1.4} = 19.3\text{ in}$

The student dissects the problem in appropriate parts and uses parts to answer the problem in this task. The student does not indicate that the answer is an approximation.

Practitioner – Sample 1 (cont.)

(1) First, I filled in 45° for each base angle of the entire triangle because it was isosceles.

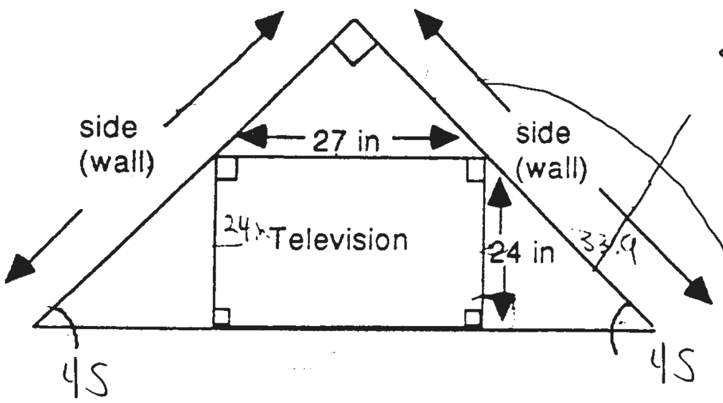
(2) I noted that the T.V. had 11 sides, then I filled in the corresponding angles of the three triangles

(3) I then used SOH CAH TOA on the triangles to the left and right of the T.V.

(4) Finally I used my knowledge of 45°-45°-90° ∠ to solve the top one, and then added the two ~~unknowns~~ unknowns

There is a clear and straightforward explanation.

Practitioner – Sample 2



SohCahToa

$\sin 45^\circ = \frac{24}{x}$

$\cancel{45x = 24}$
$\cancel{45 \quad 45}$

$.7071$

$\frac{24}{.7071} = \frac{.7071x}{.7071}$

$x = 33.4$

$\cancel{\sin 45}$
$\boxed{\cos 45^\circ = \frac{x}{27} \quad 14.09 = x}$

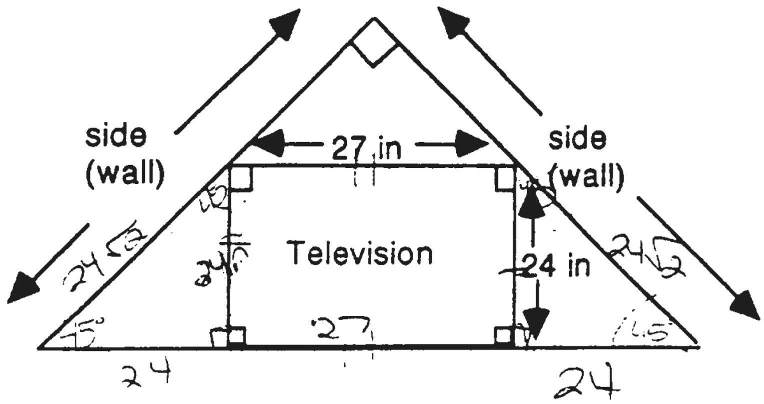
$$\begin{array}{r} 19.09 \\ +33.9 \\ \hline \boxed{52.99\text{ in}} \end{array}$$

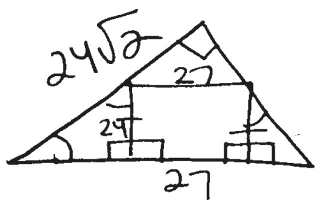
Practitioner – Sample 2 (cont.)

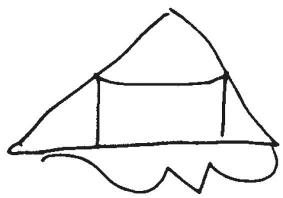
If you want the side walls to be to same them That will make the whole triangle isosoles.
I Know that I angle is 90°, which makes it a 45°-45°-48. Also I know it is not equalateral.
Also, the ~~m~~ base ∡s of all the other △s are 45°
because we know that their is a 90° angle (Given or perpendicular Lines form = 90° ∡s) and it is isosoles (Given). Now that I know all of the ∡s I can use Trig. to find the lengths.

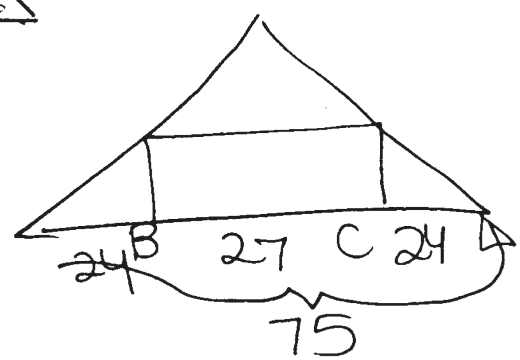
The student gives justification for use of trigonometry, but s/he Does not detail the process of the solution.

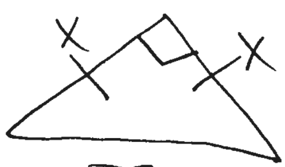
Expert – Sample 1











75
$(\sqrt{2}x)$

$\frac{75}{\sqrt{2}} = \frac{\sqrt{2}x}{\sqrt{2}}$

$\boxed{x \approx 53.03 \text{ or } \frac{75}{\sqrt{2}}}$

Each wall should be about $\boxed{53}$.00 inches
or
$\frac{75}{\sqrt{2}}$ inches (exactly)

The student indicates that the answer is an approximation.

The student approaches the problem through parts, but then switches to entire triangle.

The student shows insight by simplifying the problem.

Expert – Sample 1 (cont.)

Since we know that 2sides of the large △ are =, then it is anisosceles △. We also know that there is a rt. <, making the large △ a 45°-45°-90°, since the base <'s must be = in an isosceles △. we also know that the two smaller △'s to the sides of the T.U. are also 45°-45°-90°s because each has a rt. < (that are formed from the 90° <'s from the T.U. set) and each has an < of 45° (from the larger △) making the 3rd < a 45° We too then know that both legs of the smaller △'s are 24in because in a 45-45-90 the 45's = eachother

Then I did pythagerum thereom to figure the hypothenuse of the Smaller △'s. I figured ther hypothenusi of the large triangle by adding the long side of the T.U. and both legs of the smaller △'s to get 75in.

75 is $5\sqrt{2}x$ — so $x = \frac{75}{\sqrt{2}}$

For an Expert level we would like the student to connect the two different approaches and comment on the efficiency of each.