Making Squares

A piece of wire 63 inches long is cut into 2 parts. The 2 parts are then bent to form 2 different-size squares. The difference between the measures of the perimeters of the 2 squares is 5 inches. What is the sum of the areas of the 2 squares?

Making Squares

Common Core Task Alignments

Mathematical Practices: 1, 4, 6

Grade 6 Content Standards:
6.EE.5, 6.EE.6, 6.EE.7, 6.G.1

Grade 7 Content Standards:
7.EE.4a

Grade 8 Content Standards:
8.EE.7a, 8.EE.7b

Suggested Grade Span
Grades 6–8

Grade(s) in Which Task Was Piloted
Grade 6

Task
A piece of wire 63 inches long is cut into 2 parts. The 2 parts are then bent to form 2 different-size squares. The difference between the measures of the perimeters of the 2 squares is 5 inches. What is the sum of the areas of the 2 squares?

Alternative Versions of Task

More Accessible Version:
A piece of wire 44 inches long is cut into 2 parts. The 2 parts are then bent to form 2 different-size squares. The difference between the measures of the perimeters of the 2 squares is 4 inches. What is the sum of the areas of the 2 squares?

More Challenging Version:
Making Equilateral Triangles
A piece of wire 30 inches long is cut into 2 parts. The 2 parts are then bent to form 2 different-size equilateral triangles. The difference between the measures of the perimeters of the 2 equilateral triangles is 6 inches. What is the sum of the areas of the 2 equilateral triangles?

NCTM Content Standards and Evidence
Geometry Standard for Grades 6–8
Instructional programs from pre-kindergarten through grade 12 should enable students to:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

- ***NCTM Evidence:*** Understand relationships among the angles, side lengths, perimeters, areas and volumes of similar objects.

- ***Exemplars Task-Specific Evidence:*** This task asks students to find the lengths of squares knowing their perimeters and then finding their areas.

Number and Operations for Grades 6–8
Instructional programs from pre-kindergarten through grade 12 should enable students to ...

- Compute fluently and make reasonable estimates.

  - ***NCTM Evidence:*** Work flexibly with fractions, decimals and percents to solve problems.

  - ***Exemplars Task-Specific Evidence:*** This task requires students to work with parts of an inch using decimals or fractions.

Time/Context/Qualifiers/Tip(s) From Piloting Teacher

This is a short-length task and can be completed in one class period. Students should be familiar with finding area and perimeter of rectangles and be able to work with decimals. Calculators should be made available to students. Squaring the fractions and then finding the sum is too cumbersome.

Links

This task can be linked to design.

Common Strategies Used to Solve This Task

Many students used guess, check and refine to find the perimeters of the two squares. They then found the length of one side, squared the sides to find the area and added the two areas.

Possible Solutions

Original Version:
The smaller square measures 7.25 inches by 7.25 inches and has an area of 52.5625 square inches. The larger square measures 8.5 inches by 8.5 inches and has an area of 72.25 square inches. The total area of the two squares is 124.8125 square inches.

More Accessible Version:
The smaller square measures 5 inches by 5 inches and has an area of 25 square inches. The larger square measures 6 inches by 6 inches and has an area of 36 square inches. The total area of the two squares is 61 square inches.

More Challenging Version:
The smaller equilateral triangle measures 4 inches on each side and has an area of $4\sqrt{3}$ or approximately 6.9282032 square inches. The larger equilateral triangle measures 6 inches on each side and has an area of $9\sqrt{3}$ or approximately 15.5884573 square inches. The total area of both equilateral triangles is $13\sqrt{3}$ or approximately 22.51666 square inches.

Task-Specific Assessment Notes

General Notes: Be sure that the measurements in the student work are labeled correctly.

Novice: The Novice will fail to understand all the parameters of the problem. Although they may appear to draw two squares, there will not be enough evidence to show a successful strategy. No reasoning or justification for reasoning will be present and no mathematical connection will be made. The diagrams will not be labeled adequately.

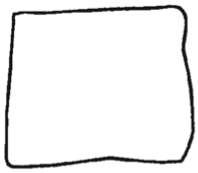
Apprentice: The Apprentice may solve only part of the task correctly, show understanding of area and perimeter but will make a mathematical error, or solve all parts of the task but fail to make a mathematical connection, observation or verify the solution. There may also be a flawed representation.

Practitioner: The Practitioner will have achieved a correct solution. There will be an understanding of all parts of the task. An understanding of area and perimeter will be demonstrated. There will be some use of mathematical representation and a mathematical connection or observation will be made.

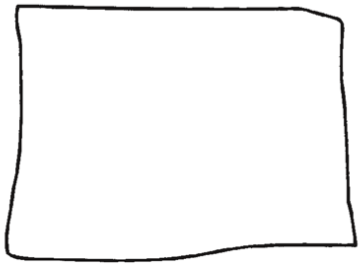
Expert: The Expert will find a correct solution. The solution will show a deep understanding of the problem. They may use formulas and/or algebra to generalize the solution. There will be mathematical connections or observations made.

Novice

There is no solution to the problem.



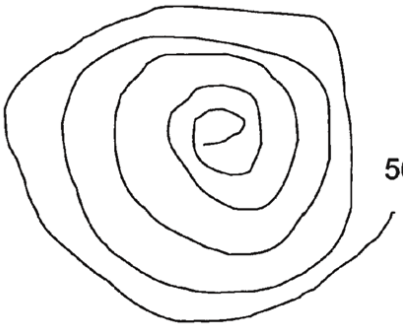
13 inches



50 inches

No reasoning or justification of reasoning is present.

13 inches



50 inches

Little or no communication of an approach is evident.

Apprentice

Making Squares

A 63 inch piece of wire is cut into two pieces to make two squares. One square has a perimeter measuring 5 inches longer than the other square. I need to find the sum in the areas of the two squares.

I took 63 and divided it by 2 to get the two pieces of wire. Each piece measures 31.5 inches. If this piece were bent to make a square each side of the square would measure 7.875in. (31.5 ÷ 4 = 7.875). Now I know that one square has a perimeter measuring 5 in more than the other so add 5 to the perimeter which will add 1.25 in to the sides (5 ÷ 4 = 1.25) and the sides of this square measures 9.125 inches (7.875 + 1.25 = 9.125).

Now that I know the size of the two squares I can find there areas by using the formula A = l x w.

Square one has sides measuring 7.875 in.

A = l x w

A = 7.875 x 7.875

A = 62.015625 square inches

Square two has sides measuring 9.125 in.

A = l x w

A = 9.125 x 9.125

A = 83.265625 square inches

The student has an approach that could have worked.

Now I can find the sum of the two areas. I will do this by adding the area of the small square to the area of the large square.

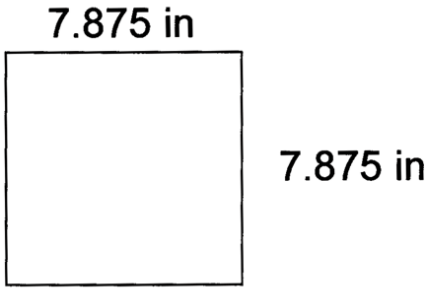
62.015625 + 83.265625 = 145.28125 square inches. This is the sum of the two areas.

Work is organized and sequenced. There is use of formal math language.

No mathematical connections are made.

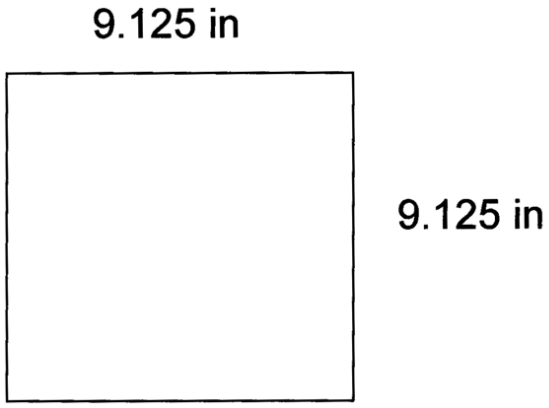
Apprentice (cont.)

Smaller Square



The answer is incorrect.

Larger Square



There is evidence of some correct mathematical reasoning and use of previous knowledge (the use of formulas for area and adding two areas).

Practitioner

A piece of wire 63 inches long is cut into two pieces and then bent to form two squares. One square has a perimeter measuring 5 inches longer than the other square. I need to find the dimensions of the two squares and then find the sum of the areas of the two squares.

Here is how I am going to solve this problem. I will make a chart to keep track of my information. I will start with a square whose sides measure one inch. Then I will find its perimeter by multiplying by 4. Subtract this perimeter from the 63 inches of wire to find the perimeter of the second square. I will check the dimensions to see if the perimeters differ by 5 inches. Then I will divide these perimeters by 4 to find the lengths of the sides of the two squares. When I have this I have the dimensions of the two squares that I need, I will computer the areas and find the sum.

I did my chart using multiples of 4 in the first column because the formula for perimeters of squares is 4s and this is a multiple of 4. When I got a difference in perimeter of 1 I new that the value had to be between 28 and 32 and probably closer to 28 so I tried 29 and that worked.29 can still be a multiple of 4 as long as one of the factors is a fraction. There is only one set of squares that have a perimeter 5 inches apart, These squares measure 7.25 in by 7.25 in, and 8.5 in by 8.5 in, The information in the chart says that the two squares have perimeters of 29 inches and 34 inches. This means that the wire must be cut into two pieces measuring 29 inches and 34 inches (29 + 34 = 63 inches).

Now I can find the areas of the two squares by using the formula $A = s^2$, s represents the measure of the side of the square.

Square 1	Square 2
side measures 7.25 in	side measures 8.5 in
$A = 7.25^2$	$A = 8.5^2$
$A = 52.5625$ in^2	$A = 72.25$ in^2

The sum in areas is $52.5625 + 72.25 = 124.8125$ in^2

The student uses exponents appropriately in the area formula.

Formal and precise mathematical language is used throughout, such as *multiple*, *factor*, *dimensions* and *formula*.

The student's strategy—making a chart, increasing the length of the sides of the squares, checking to see if perimeters differ by five, and then summing the areas—worked.

Practitioner (cont.)

This problem is like those that we had to find the largest area for a pet if we knew the amount of fencing that could be used. In both types of problems the perimeter is known and you have to find the lengths of the sides. The pet problems usually look at more than squares, You can find the sides of rectangles. However, the largest area is when you have a square.

Mathematical observations are made.

Square 1		Square 2		Difference in perimeter measures (in)
Perimeter (in)	Side (in)	Perimeter (in)	Side (in)	
4	1	59	14.75	55
8	2	55	13.75	47
12	3	51	12.75	39
16	4	47	11.75	31
20	5	43	10.75	23
24	6	39	9.75	15
28	7	35	8.75	7
32	8	31	7.75	1
29	7.25	34	8.5	5

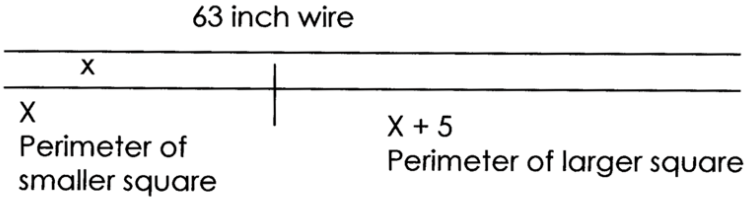
The student connects this task to similar tasks and compares and contrasts them.

The student uses a systematic approach and has a well-organized presentation.

Expert

A piece of wire 63 inches long is cut into two pieces and then bent to form two squares. One square has a perimeter measuring 5 inches longer than the other square. I need to find the dimensions of the two squares and then find the sum of the areas of the two squares.

I will use an variable and equation to help me solve this problem.



x represent the perimeter of the smaller square.
x + 5 represents the perimeter of the larger square
perimeter of larger – perimeter of smaller = 5
x + x + 5 = 63
x + x + 5 = 58 + 5
x + x = 58
x + x = 29 + 29
x = 29
x + 5 = 34
The perimeter of the smaller square is 29 inches and the perimeter of the larger square is 34 inches.

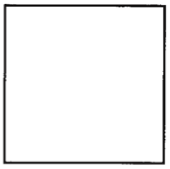
The student's use of an equation is an efficient strategy.

Precise and symbolic mathematical language is used throughout.

Expert (cont.)

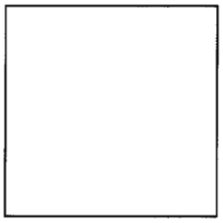
A correct answer is achieved.

Smaller Square
Perimeter 29 in
Side = 7.25 in



x

Larger Square
Perimeter 34 in
Side = 8.5 in



To find the length of the sides of the square divide the perimeter by 4.
29/4 = 7.25 inches 34/4= 8.5 inches

Now I can find the areas of the two squares by using the formula $A = s^2$, s represents the measure of the side of the square.

Smaller Square
side measures 7.25 in
$A = 7.25^2$
$A = 52.5625 \text{ in}^2$

Larger Square
side measures 8.5 in
$A = 8.5^2$
$A = 72.25 \text{ in}^2$

The sum in areas is $52.5625 + 72.25 = 124.8125 \text{ in}^2$

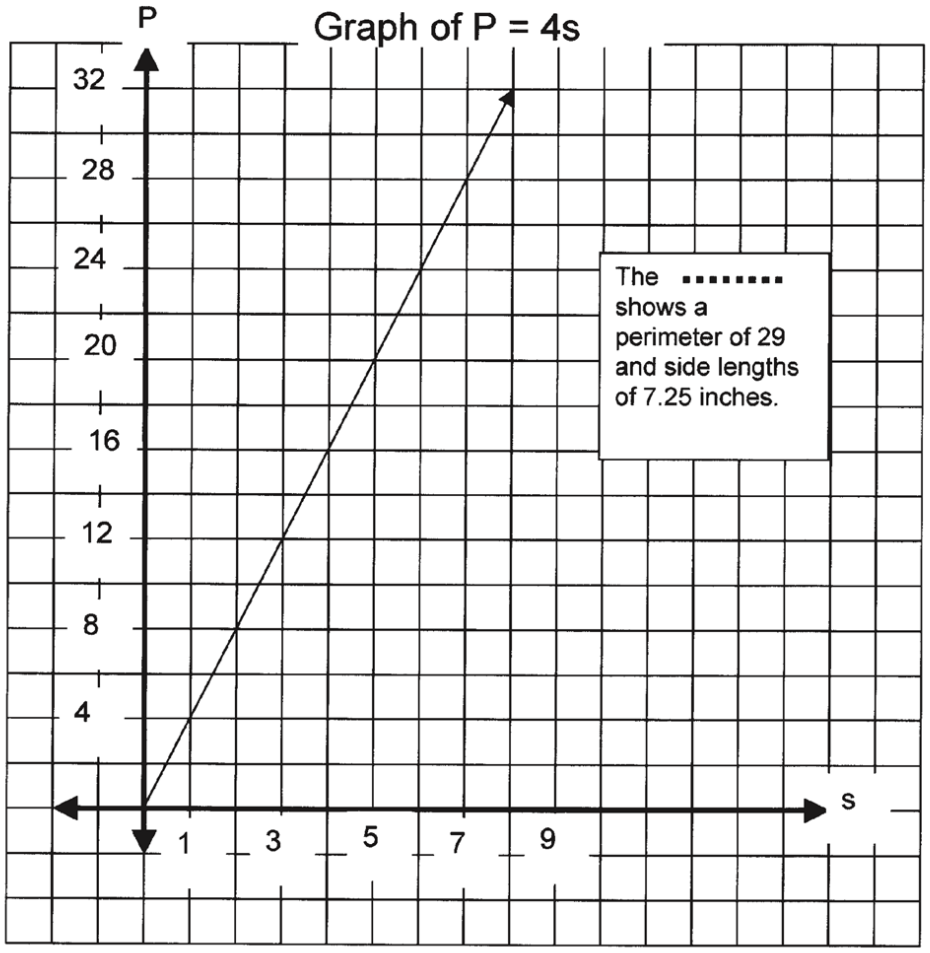
Here are some things that I noticed about this problem. I was surprised to see that the perimeters were 29 inches and 34 inches. I thought that the perimeters were going to be multiples of 4 because the formula for the perimeter of squares is 4s where s represents the measure of the side of the square and that represents a multiple of 4. I just do not think of a decimal times 4 as being a multiple of 4. However if you graph p = 4s then these values will appear on the graph. (see graph)

This problem was similar to other problems that we have done where we either kept the perimeter or area constant. However, in those problems we could consider rectangles as well as squares. In this problem we had to keep the sum of the perimeters constant. That made the approach change a bit.

A strong mathematical connection is made.

Mathematical connections are made to other problems.

Expert (cont.)



The graph is appropriate for the earlier discussion of P = 4s.